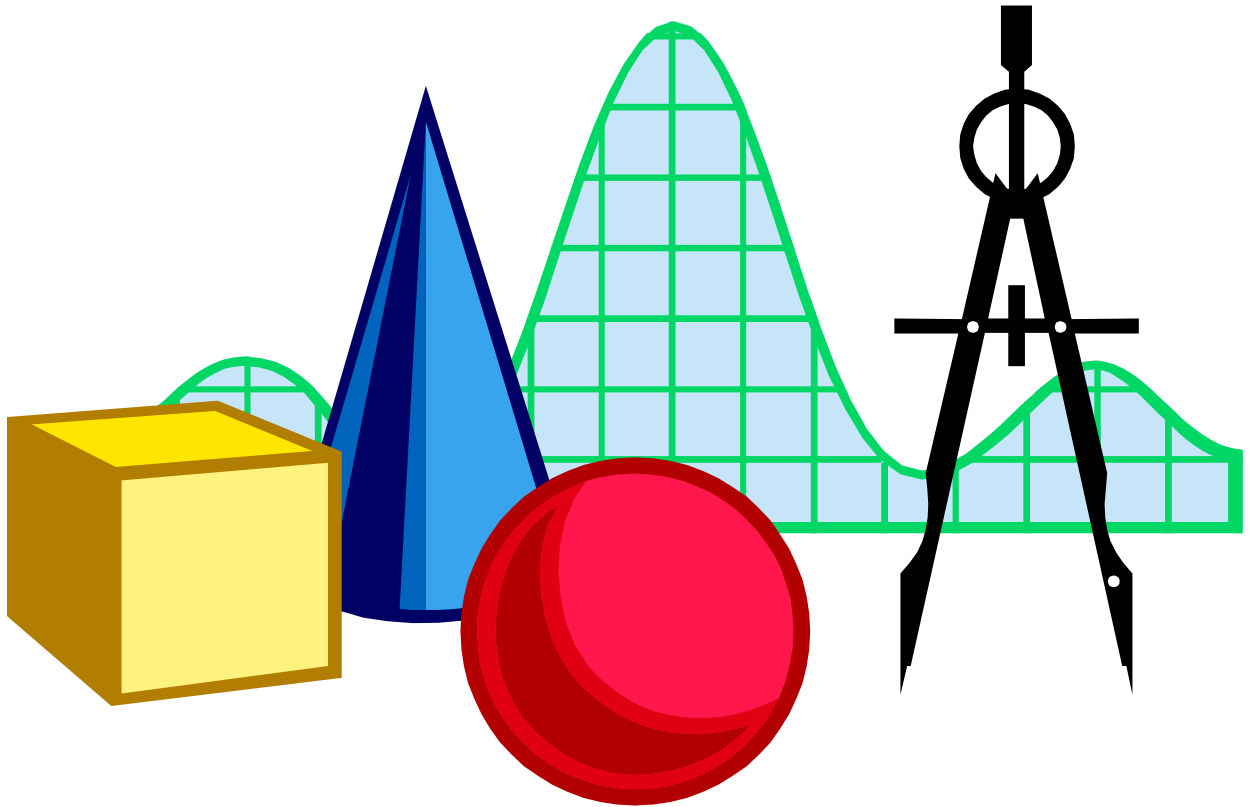


# ***Tulsa COUNTS!***



## **Tulsa Public Schools Mathematics Initiative**

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**June, 2003**

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Tulsa Public Schools Mathematics Initiative

**Foreword**

**David E. Sawyer, Ed. D.**

I remember telling one of my students many years ago, while teaching a statistics course for teachers, not to worry when she explained her nervousness about the class—a requirement for her Masters degree. I asked her if she could add, subtract, multiply and divide. After receiving an affirmative and confident response, I told her she would be all right. Our students in our classrooms today need to likewise be “all right” when it comes to these basic skills. As the quote in the introduction to this publication correctly states, “Mathematics is a vital academic skill and a critical part of academic learning.”

Adding, subtracting, multiplying and dividing are skills that are absolutely essential. Schools are where these skills are developed and nurtured. Classrooms are where they are introduced, practiced and perfected. Teachers make the difference for the students. From basic arithmetic to calculus and beyond, we must provide the experiences necessary to support and encourage deeper understanding and improved opportunities for all students. Like reading, mathematical skills are basic building blocks, together creating stair steps to the future. Some will aspire to more difficult and challenging mathematics coursework as they grow and mature, but all must master the basics or risk a dismal prospect for personal and/or professional success.

***Tulsa COUNTS!*** is our plan to assure this objective is met. I commend it to you and encourage you as you assist our students in attaining excellent results. Best wishes.

## INTRODUCTION

Mathematics is a vital academic skill and a critical part of academic learning (R. Anderson, 1996). Teaching children to understand and use mathematics must be a high priority (Riley, 1998). The need to improve preparation of mathematics educators and mathematics instruction is widely recognized by the National Council of Teachers of Mathematics. The controversy arises in determining how best to achieve these desirable outcomes.

Mathematics curriculum debates divide proponents into two groups. The first group supports emphasis on basic mathematics skills such as memorizing multiplication facts and learning algorithms. The second group emphasizes understanding concepts such as how multiplication relates to addition and developing their own algorithms (*The Great Math Debate, CER Monthly Letter*, Oct./Nov. 1999). Advocates of basic skills emphasize learning rules and procedures; those who stress understanding are interested more in the process of mathematics than in its content. These descriptions oversimplify both positions. Nonetheless, neither approach, alone, would prepare mathematics students in the United States to be successful in a global economy that requires mathematical literacy.

All students can learn mathematics and they deserve the opportunity to do so. It is essential that students acquire the necessary skills to compute on grade-level by the end of third grade and maintain grade-level performance through school completion. Further, all students must be engaged in developing basic skills, problem-solving skills, and advanced conceptual understanding through the use of standards-based materials. The research on mathematics education tells us that children who do well in mathematics from the beginning rarely stumble later (J. Kilpatrick, J. Swafford, & B. Findell, 2001). Conversely, students who fall behind in mathematics in the primary grades struggle to recover. Children who are not proficient in basic mathematics skills and are not competent problem solvers will not have access to advanced mathematics courses. The call to action is clear. Intensive, systematic mathematics intervention must be available to all children, regardless of grade level, who are experiencing difficulties with mathematics.

Effective mathematics instruction must be a reality for every child. Results from the 1999 Third International Mathematics and Science Study (TIMMS) survey of eighth-grade mathematics teaching among seven countries revealed that in the relatively high-achieving countries, a variety of methods were employed, rather than a single, shared approach to the teaching of mathematics (J. Hiebert, 1999). The TIMMS report ranked the mathematics performance of students in the United States near the bottom when compared to the performance of students in other countries. The TIMMS report describes the United States mathematics curriculum as unfocused--a mile wide and an inch deep. More topics are included in the United States curriculum at each grade level than in the curricula of most other countries (Edward A. Silver, 1998). According to the report, too many topics are covered in one year and few topics are developed in enough depth to achieve mastery.

## **PURPOSE**

*Tulsa COUNTS!* is a comprehensive mathematics initiative that uses a standards-based curriculum to ensure all students develop basic mathematics skills, advanced conceptual understanding, as well as problem solving skills included in the *Tulsa Model for School Improvement*.

## **PRIORITIES**

*Tulsa COUNTS!* addresses the following district-wide priorities as identified in the *Tulsa Model for School Improvement*:

- Improve the academic performance of all students,
- Standardize the comprehensive mathematics program to meet the needs of a mobile and diverse student population,
- Standardize assessment as part of the comprehensive mathematics program,
- Provide teacher training grounded in research-based best practices,
- Promote parental involvement and community support.

## **STANDARDS AND RESEARCH-BASED BEST PRACTICES**

The use of standards-based best practices in mathematics instruction is supported in the research on mathematics teacher preparation (L. Ma, 1999) as well as in the research literature on mathematics education (Goldsmith, & Kantrov, 1998). Represented in every phase of *Tulsa COUNTS!* is a full range of professional standards, research-supported assumptions, and documented best practices.

### **Standards**

The core of *Tulsa COUNTS!* is Oklahoma's *Priority Academic Student Skills (PASS)* adopted in August 2002. As a comprehensive mathematics initiative, *Tulsa COUNTS!* addresses the needs faced by mathematics teachers as reported by the National Council of Teachers of Mathematics (NCTM), the National Science Board (NSB), and the National Assessment of Educational Progress (NAEP). These reports consistently stress:

- Problem solving should be the focus of school mathematics.
- The study of mathematics should emphasize developing higher order thinking skills (reasoning, analyzing, estimating, inferring, and others), understanding of concepts, communicating about mathematics, making mathematical connections, and applying mathematics.
- Basic skills in mathematics should be defined to include more than computational facility.

- School mathematics should provide for an integrated study with increased emphasis on content such as geometry, measurement, patterns, relations, numeration, probability, statistics, logic, algorithmic thinking, and applications.
- Mathematics programs should take advantage of technology such as calculators and computers.

### **Research**

Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding (J. Hiebert, & T. Carpenter 1992). To understand what they learn, they must be active learners who enact for themselves the verbs that permeate the mathematics curriculum. This happens most readily when students work in groups, engage in discussion, make presentations, and take charge of their own learning (W. Stroup, 1999). Problem solving, rather than being a distinct topic to be covered, becomes the context in which all concepts and procedures are learned. When students truly solve problems, they engage in authentic tasks that they do not know how to solve in advance (J. W. Stigler, and J. Hiebert, 1999). Students engaged in problem solving construct understanding through the processes of assimilation and accommodation with prior knowledge. Instruction that develops mathematical proficiency is neither simple, common, nor well understood. It comes in many forms. The best approach is to modify instruction to adapt to the goal of the lesson. Sometimes a concept is best understood by using direct instruction, or by sometimes having students work with a partner or a team to discover the meaning of complicated material.

The implementation of *Tulsa COUNTS!* addresses these important issues, meets the needs of all students, and gives them the tools to be competitive in a rapidly changing world. Developed by a district-wide team of mathematics educators, this initiative represents systemic change in mathematics education.

### **Best Practices**

Zemelman, Daniels, and Hyde (1998), observe that summary reports, meta-analyses of instructional research, bulletins, and professional recommendations provide a wealth of documented best practices. These recommendations support state of the art teaching in every critical field of classroom instruction, including mathematics. They summarize and categorize these practices into the following two lists:

#### **THE MORE LIST**

- More experiential, inductive, hands-on learning
- More active learning with all the

#### **THE LESS LIST**

- Less whole-class directed instruction.
- Less student passivity; for example, less sitting, listening, and receiving

attendant noise of students doing talking, and collaborating

- More deep study of a smaller number of topics
- More responsibility transferred to students for their work: goal-setting, record-keeping, monitoring, and evaluation
- More choices for students: for example, choices on projects and demonstrations
- More attention to effective needs and varying learning styles of students
- More cooperative, collaborative activity
- Less student time reading textbooks
- Fewer attempts by teachers to cover large amounts of material
- Less rote memorization of facts and details
- Less stress on competition and grades
- Less reliance on standardized tests as the ONLY measure of student learning

## BACKGROUND

*Tulsa COUNTS!* uses standards-based curriculum to ensure all students develop basic mathematics skills, problem-solving skills, and advanced conceptual understandings. To achieve these outcomes, however, four critical components of mathematics instruction should be present at all grade levels:

1. Contextual and Applied Activities
2. Mathematics Tools
3. Flexible Grouping
4. Communication

### Contextual and Applied Activities

Applied activities, also referred to as authentic projects, allow students to integrate their conceptual knowledge with their procedural knowledge, thus allowing them to apply this combined knowledge to real-world problems (Caine & Caine, 1994). For example, when students prepare for a class camping trip, they might make predictions about the cost of the items on their grocery list. After shopping, they would compare their estimations to the actual cost of the supplies. Using the grocery store receipts, they would be able to create a budget for their next trip. It is important for students to focus not just on learning mathematics but, more importantly, to focus on using mathematics in their lives. According to Nunley, (1999), a “being there” experience will allow the brain to connect new information to prior knowledge. This allows the student to move information to long-term memory.

The use of technology is another avenue for incorporating contextual and applied activities in the instructional process (Sheets, 1993). Using technology in mathematics instruction increases accuracy and speed in data collection and graphing, real-time visualization, and interactive modeling of invisible mathematical processes. Also, technology enriches mathematics instruction by supporting the students’ ability to collect, compute, and analyze large volumes of data, collaborate on data collection and interpretation, and present more varied results (Boers-van Oosterum, 1990). Further, technology can support investigation by students in every area of mathematics including geometry, statistics, algebra, measurement, and number. Through the appropriate use of technology, students can learn more mathematics more deeply (Y. M. Germain, 1991).

### Mathematical Tools

When students use tangible objects to construct the meaning of complex mathematical concepts, they develop a deeper understanding of the content. This allows them to apply these concepts accurately in a variety of new settings (Sowell, E. J., 1989). Too often, children are simply taught math *facts* without learning the tools to apply those facts (McClain, Kay, Paul Cobb, and Janet Bowers, 1998). For example, they learn the multiplication tables without being given the opportunity to really explore the *meaning* of

multiplication. Mathematical tools, referred to as manipulatives, help students develop the conceptual understanding that sometimes has been omitted from the curriculum, particularly in the lower grades. However, hands-on materials do not lose their value in the upper grades. In fact, as concepts become more complex, these materials can help to accommodate the various learning styles of students so that they are more likely to internalize the concepts (Sowell, 1989).

One way to allow students to demonstrate how to use mathematics in practical situations is to encourage the use of electronic technologies. Calculators and computers are essential tools for teaching, learning, and using mathematics. These devices furnish visual images of mathematical ideas, facilitate organizing and analyzing data, and compute efficiently and accurately (Dunham and Dick, 1994).

### **Flexible Grouping for Differentiated Instruction**

When discussing the notion of flexible grouping, two possible configurations emerge, flexible grouping within classrooms and between classrooms.

(Tomlinson, 1999) suggests that flexible grouping within a classroom responds to the needs of all learners because it allows children to work independently, with their peers, one-on-one with the teacher, or in small groups with the teacher. Rather than assigning mathematical problems solely on the basis of students' competency with a given concept, Tomlinson contends that flexible groups allow students to articulate their own understanding of the content and to learn from each other.

Working with flexible groups also allows the teacher to differentiate instruction and provide individual feedback and direction. Working one-on-one or with a small group of students, the teacher can evaluate students' conceptual and procedural abilities. The teacher is able to provide timely individualized instruction and direct students toward manipulatives and exercises that will help them build their skills. Through conferences, the teacher can also evaluate whether or not a student understands and can apply a given mathematical concept. If a student is struggling, the teacher is available to provide immediate support and reinforcement. These informal assessments provide a window into the progress of the entire class, thus shaping instruction for future lessons, activities, and projects.

(Oakes, 1985) speaks to the benefits of flexible grouping as compared to the tracking system that attempts to group students homogeneously for instruction. According to Oakes, when students are grouped by ability, more often than not, the higher group will receive a richer curriculum and more effective teachers than the others. Through flexible grouping, all students receive the type of instruction that will best prepare them for success.

## **Communication**

Effective mathematics instruction includes complex, open-ended problems in which multiple approaches and answers are possible (A. Porter, 1998). Teachers must ask complex questions to extend student thinking and students must articulate their findings and logic behind their processes. Small-group work, writing assignments and journaling give students an opportunity to clarify their own thinking and to synthesize what they have learned (Billmeyer & Barton, 1998).

The four critical components referenced above: Contextual and Applied Activities, Mathematical Tools, Flexible Grouping for Differentiated Instruction, and Communication will permeate mathematics instruction at all grade levels. Although emphasis and applications of various components will be different at each grade level, every teacher will be expected to incorporate all components for all students each year.

***Tulsa COUNTS!* - THE TULSA PUBLIC SCHOOLS  
MATHEMATICS INITIATIVE  
Pre-Kindergarten through Grade 12**

*Employees in all fields of work have issued the same request across the country: send us employees who can think, reason, and solve problems. Children must be helped to learn mathematics in a better way than we were, so that mathematical limits do not shut them out of certain life choices and career options. (Burns, 1998, p. xi)*

The ***TULSA COUNTS!*** mathematics initiative recognizes that effective Pre-K through Grade 12 mathematics instruction should be vertically integrated, beginning with grade appropriate rudimentary basic skills in the early years and building toward mastery of more complex skills, problem-solving and conceptual understanding in the latter years. Four grade-related stages have been identified for establishing instructional goals and standards attainments in mathematics.

**Pre-Kindergarten through Grade Two**

The concepts and skills related to numbers and operations are a major emphasis of mathematics instruction in pre-kindergarten through grade two. Over this span, the small child who holds up two fingers in response to the question "How many is two?" grows to become the second grader who solves sophisticated problems using multi-digit computation strategies. In these years, children's understanding of number develops significantly. During the early years, teachers must help students strengthen their sense of number, moving from the initial development of basic counting techniques to more-sophisticated understandings of the size of numbers, number relationships, patterns, operations, and place value.

Students' work with numbers will be connected to their work with other mathematics topics. For example, computational fluency (having and using efficient and accurate methods for computing) can both enable and be enabled by students' investigations of data. Knowledge of patterns supports the development of skip-counting and algebraic thinking, and experiences with shape, space, and number help students develop estimation skills related to quantity and size.

As they work with numbers, students will develop efficient and accurate strategies that they understand, whether they are learning the basic addition and subtraction number combinations or computing with multi-digit numbers. They will explore numbers into the hundreds and solve problems with a particular focus on two-digit numbers. Although good judgment must be used about which numbers are important for students of a certain age to explore, teachers will be careful not to underestimate what young students can learn about numbers. Students are often surprisingly adept when they encounter numbers, even large numbers, in problem contexts. Therefore, teachers will regularly encourage students to demonstrate and deepen their understanding of numbers and operations by solving interesting, contextualized problems, and by discussing the representations and strategies they use.

It is imperative; therefore, to provide all students with high-quality programs that include significant mathematics presented in a manner that respects both mathematics and the nature of young children. These programs will build on and extend students' intuitive and informal mathematics knowledge. They will be grounded in the knowledge of child development and take place in environments that encourage students to be active learners and accept new challenges. They will develop a strong conceptual framework while encouraging and developing students' skills and their natural inclination to solve problems. Number activities oriented toward problem solving can be successful even with very young children and can develop not only counting and number abilities but also such reasoning abilities as classifying and ordering (Clements, 1984). Recent research has confirmed that an appropriate curriculum strengthens the development of young students' knowledge of numbers and geometry (Griffin and Case, 1997; Klein, Starkey, and Wakeley, 1999; Razel and Eylon, 1991; *Principles and Standards of School Mathematics, 2000.*)

### **Grades Three through Five**

By the end of this grade band, students will be computing fluently with whole numbers. *Computational fluency* refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. The computational methods that a student uses will be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division, and number relationships. Research suggests that by solving problems that require calculation, students develop methods for computing and also learn more about operations and properties (McClain, Cobb, and Bowers, 1998; Schifter, 1999).

As students move from third to fifth grade, they will consolidate and practice a small number of computational algorithms for addition, subtraction, multiplication, and division that they understand well and can use routinely. Many students enter grade three with methods for adding and subtracting numbers. In grades three through five, they will extend these methods to adding and subtracting larger numbers and learn to record their work systematically and clearly. Having access to more than one method for each operation allows students to choose an approach that best fits the numbers in a particular problem.

Teachers in grades three through five make decisions every day that influence their students' opportunities to learn and the quality of that learning. The classroom environment they create, the attention to various topics of mathematics, and the tools they and their students use to explore mathematical ideas are all important in helping students in grades three through five gain increased mathematical maturity. In these grades, teachers will help students learn to work together as part of building a mathematical community of learners. In such a community, students' ideas are valued and serve as a source of learning, mistakes are seen not as dead ends but rather as potential avenues for learning, and ideas are valued

because they are mathematically sound rather than because they are argued strongly or proposed by a particular individual (Hiebert et al., 1997). A classroom environment that supports the learning of mathematics with meaning will have several characteristics: students feel comfortable making and correcting mistakes; rewards are given for sustained effort and progress, not the number of problems completed; and students think through and explain their solutions instead of seeking or trying to recollect the "right" answer or method (Cobb et al., 1988). Creating a classroom environment that fosters mathematics as sense making requires the careful attention of the teacher. The teacher establishes the model for classroom discussion, making explicit what counts as a convincing mathematical argument. The teacher also lays the groundwork for students to be respectful listeners, valuing and learning from one another's ideas. (*Principles and Standards for School Mathematics, 2000.*)

### **Grades Six through Eight**

In the middle grades, students will continue to refine their understandings of addition, subtraction, multiplication, and division as they use these operations with fractions, decimals, percentages, and integers. Teachers will be attentive to conceptual obstacles many students encounter as they make the transition from operations with whole numbers to fractional parts. The continued development and improvement of mathematics skills identified for elementary students will be emphasized for secondary students using increasingly difficult materials.

Mathematics instruction must be incorporated into the curriculum in order to increase students' mathematics proficiency. Emphasis must be on using mathematics to solve problems as well as on mathematics as an enjoyable activity. Students must be provided strategies for solving mathematics problems including text preview, prior knowledge activities, vocabulary development, and graphic organizers. The overriding goal behind any mathematics problem-solving activity should be to foster active, engaged, independent thinkers.

It is recommended that middle grades mathematics students learn significant amounts of algebra and geometry throughout grades six, seven, and eight. Students will see algebra and geometry as interconnected with each other and with other content areas in the curriculum. They also will relate algebraic and geometric ideas to other topics; for example, when they reason about percents using visual models or equations or when they represent an approximate line of fit for a scatter plot both geometrically and algebraically. Students can gain a deeper understanding of proportionality if it develops along with foundational algebraic ideas such as linear relationships and geometric ideas such as similarity. (*Principles and Standards of School Mathematics, 2000.*)

### **Grades Nine through Twelve**

Whereas middle grade students were introduced to irrational numbers, high school students should develop an understanding of the system of real and complex numbers. High school

students will understand that irrational numbers can only be approximated with fractions or by terminating or repeating decimals. They will understand the difference between rational and irrational numbers. High school students' understanding of irrational numbers needs to extend beyond  $\pi$  and  $\sqrt{2}$ . They also need to understand the development of the imaginary numbers and their use in science exploration.

High school students will be able to use their understanding of numbers to explore new systems such as vectors and matrices. By working with examples that include forces and velocities, students can learn to appreciate vectors as a means of simultaneously representing magnitude and direction. Using matrices, students can also see connections among major strands of mathematics. For example, they can use matrices to solve systems of linear equations, to represent geometric transformations (some of which can involve creating computer graphics), and to represent and analyze vertex-edge graphs. High school students will be able to decide whether a problem calls for a rough estimate, an approximation to an appropriate degree of precision, or an exact answer. More specifically, students will be able to select a suitable method for computing from among mental mathematics, paper-pencil computations, and the use of calculators or computers, and should be proficient with each method.

In high school, students will build on their prior knowledge, learning more varied and more sophisticated problem-solving techniques. Students will increase their abilities to visualize, describe and analyze situations in mathematical terms. These students will learn to use a wide range of explicitly and recursively defined functions to model the world around them. Moreover, their understanding of the properties of those functions should give them insights into the phenomena being modeled. High school students will understand statistics and probability sufficiently to provide them ways to think critically about a wide range of issues that have important social implications.

Students need to develop increased abilities for justifying claims, proving conjectures, and using symbols in reasoning. They will be expected to learn to provide carefully reasoned arguments in support of their claims. Furthermore, they will be given practice making and interpreting oral and written claims so that they can communicate effectively while working with others and can convey the results of their work with clarity and power. Students will continue to develop facility with such technological tools as electronic spreadsheets, data-gathering devices, computer algebra systems, and graphing utilities that enable them to solve problems that would require large amounts of computational time if done by hand (Steen, 1997). This facility with technological tools also helps students analyze data.

## ***Tulsa COUNTS! and The Tulsa Model for School Improvement*** **Planning for Effective Mathematics Instruction**

The district model for improving student achievement includes a focus on improving student achievement in mathematics. *The Tulsa Model for School Improvement* along with ***Tulsa COUNTS!*** provide a comprehensive conceptual framework for achieving this goal. Each school will include in their site plan research-based intervention strategies that address the following eight skill areas:

### **Critical Skill Areas**

- ***Assessment***  
Teachers should incorporate various assessment strategies determining the skill level of their students. The use of projects, group tests, journals, oral presentations, technology, and career exploration should complement pencil and paper assessments.
- ***Classroom Management Skills***  
Included in classroom management skills is the utilization of procedures, cooperative groups, and student-centered activities. Organizational skills are essential in the implementation of hands-on, problem-based mathematics. The teaching environment is critical to the learning process.
- ***Teaching for Higher Order Thinking***  
Learning basic skills is necessary to achieve higher order thinking skills, however, that should not be the only focus. Application of the basic skills will help develop higher order thinking skills and improve student achievement. Connecting information in real-world situations through inquiry is the recommended method for achieving higher order thinking skills. (Refer to *Questioning Strategies* in the Appendix).
- ***Instructional Strategies***  
In order to affect the learning of mathematics, various teaching strategies, such as identifying similarities and differences, cooperative learning, generating and testing hypotheses, and questions, cues and advanced organizers must be utilized.
- ***Communicating Content/Concepts***  
Teachers need a deep understanding of the mathematics they teach, i.e., concepts, practice, principles, representation, and applications to support effective instruction. A teacher's conceptual understanding of mathematics can affect classroom instruction in a direct and positive way.
- ***Development and Use of Conceptual Curriculum, Standards, and Frameworks***  
Standards-based curriculum writing that incorporates big ideas and conceptual frameworks is a key ingredient for developing a site plan.
- ***Professional Learning Communities***  
Teachers engage in continuous improvement and professional learning collaboration on issues concerning their assigned tasks.
- ***Teaching for Equity***  
The learning styles of under-represented groups, such as females and some minorities, should be studied and disaggregated test data should be analyzed to plan for success.

## ***TULSA MODEL FOR SCHOOL IMPROVEMENT*** **STANDARDIZATION OF THE MATHEMATICS COMPONENTS**

From exploring ideas related to patterns, shapes, numbers, and space in the earliest stages of Pre-Kindergarten mathematics instruction through understanding of the system of real numbers at the high school level, *Tulsa COUNTS!* holds high expectations for student achievement in mathematics at every grade level. The *Tulsa COUNTS!* initiative places substantial expectations on teachers and learners alike. These expectations will translate into the exemplary teaching and learning that is necessary to attain full quantitative literacy.

*Tulsa COUNTS!* proposes a standard format for development of the mathematics program at each school site. School plans will be consistent across grade bands. All schools will standardize on the following components:

### **ELEMENTARY MATHEMATICS**

- Implement all components of the *Tulsa Model for School Improvement*.
- Incorporate research based math intervention strategies in the math goal of the site plan (see Appendix).
- Schedule mathematics instruction for a minimum of one hour daily.
- Provide mathematics materials for practice which contain at least one of the following elements: hands-on investigations, problem solving activities, mathematical communication, connections between disciplines and the real world, and reasoning skills.
- Provide *Connect to the Standards* training for all kindergarten through fifth grade teachers.
- Study *ED Thoughts – What We Know About Mathematics Teaching and Learning* as a staff.
- Use multiple, ongoing assessments to diagnose difficulties and quarterly benchmark tests to plan appropriate instruction and intervention.
- Provide additional in-school and/or extended time instruction for students who need extra support.
- Provide instruction in flexible, cooperative groups or pairs for problem-solving investigations, as well as engagement in whole class instruction for concept and comprehension development.
- Include basic skills instruction along with problem-based instruction.
- Provide materials, recommendations, and training for parents to engage in at-home activities.
- Provide appropriate training for mentors and volunteers in the classroom.
- Integrate technology into the mathematics curriculum.

## SECONDARY MATHEMATICS

- Implement all components of the *Tulsa Model for School Improvement*.
- Incorporate research-based math intervention strategies in the math goal of the site plan (see Appendix).
- Study *ED Thoughts – What We Know About Mathematics Teaching and Learning* as a math department and site instructional team.
- Integrate mathematics activities across the curriculum.
- Provide professional development in research-based strategies for teaching mathematics.
- Provide *Connect to the Standards* training for all teachers, grades sixth through eighth grades.
- Schedule daily independent practice for students.
- Use multiple, ongoing assessments to diagnose difficulties and quarterly benchmark tests to plan appropriate instruction and intervention.
- Develop an Individual Plan for students performing basic skills below grade level and provide extended time for students not meeting state standards.
- Provide materials, recommendations, and training for parents to engage in at-home activities.
- Schedule periodic reflective sessions with instructional staff to monitor and adjust the mathematics program.
- Integrate technology into the mathematics curriculum.

**STANDARDIZED EVALUATION AND ASSESSMENT PLAN**  
**Pre-Kindergarten through Grade 12**

The true value of any initiative is reflected in the efficacy of its formal and informal assessment components. *Tulsa COUNTS!* requires the development or acquisition of substantial and authentic assessment tools and instruments. All students in Tulsa Public Schools will be assessed by multiple on-going assessments for the acquisition of mathematics knowledge and competencies for the grade level in which they are enrolled. Such assessments are vital to planning appropriate instruction. Assessments will also be used at each site to monitor the effectiveness of the school’s mathematics program and to make changes in instructional focus and methods as necessary. These assessments should be both formal and informal. For example, in addition to the more formal, standardized assessments, all schools will use a variety of informal tests that may include observation, group tests, teacher/student conferences, portfolios, end-of-unit tests, and teacher-made tests. Assessment tools are currently being identified and developed.

*Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989, p.191) makes the following general recommendations for changing student assessment:

<b>Increased Attention</b>	<b>Decreased Attention</b>
<ul style="list-style-type: none"> <li>• Assessing what students know and how they think. Having assessment be an integral part of teaching</li> <li>• Focusing on a broad range of mathematical tasks and taking a holistic view of mathematics</li> <li>• Developing problem situations that require the application of a number of mathematical ideas</li> <li>• Using multiple assessment techniques, including written, oral, and demonstration formats</li> <li>• Using calculators, computers, and manipulatives in assessment</li> </ul>	<ul style="list-style-type: none"> <li>• Assessing what students do not know about mathematics</li> <li>• Having assessment to be simply counting correct answers on tests for the sole purpose of assigning grades</li> <li>• Focusing on a large number of specific and isolated skills</li> <li>• Using exercises or word problems requiring only one or two skills</li> <li>• Using only written tests</li> <li>• Excluding calculators, computers, and manipulatives from the assessment process</li> </ul>

## COMPREHENSIVE MATHEMATICS PROGRAMS CURRENTLY UNDER REVIEW

- ***Growing with Mathematics*** (Wright Group/McGraw-Hill, Grades Pre-K-6)  
*Growing with Mathematics* draws its philosophy from current thinking about the teaching of mathematics. Research tells us that to become good problem solvers, students must develop a strong foundation of mathematical concepts and a wide range of skills and strategies. This foundation includes a thorough knowledge of number facts, confidence in mental computation, and the ability to estimate. The range of components in *Growing with Mathematics* supports a balanced approach and features real-world situations designed to stimulate students' interest and involvement in mathematics. Innovative materials such as the *Discussion Book*, the eight *Mathtales* titles, and the *Mathematics from Many Cultures* series provide the setting for an exploration of key mathematical concepts.

It is universally agreed that skills continue to be an important part of any mathematics program. Research shows that skills are long lasting when students have developed them through the use of thinking strategies, rather than rote learning. *Growing with Mathematics* supports the use of strategies and the development of skills in the following ways:

- Carefully sequenced activities,
  - Easy-to-manage materials that promote visual thinking ,
  - Ongoing reinforcement of skills through *Number Sense* activities, *Maintaining Concepts and Skills* black line masters, and the *Practice and Homework Book*,
  - Regular, thorough assessment.
- ***Investigations in Number, Data, and Space*** (Scott Foresman, Grades K-6)  
*Investigations in Number, Data, and Space* reflects the view that mathematics in elementary school includes arithmetic and more. In the elementary grades, students need to develop a foundation in several key content areas of mathematics, such as number, data, and geometry. The *Investigations* curriculum also includes activities based on recent research on young children's understanding of the mathematics of change.

Each *Investigations* unit offers from two to eight weeks of mathematical work on topics in number, data analysis, and geometry; the number of units per year varies by grade level. Because of the many interconnections among mathematical ideas, units may revolve around two or three related areas; for example, addition and subtraction or geometry and fractions.

In each unit, students explore the central topics in depth through a series of investigations, gradually encountering and using many important mathematical ideas. Rather than working through a textbook or workbook doing page-by-page exercises, students actively engage in mathematical thinking to solve larger mathematical problems.

Students represent their thinking and use mathematical tools and appropriate technology, including calculators, as a natural part of their work. They work in a variety of groupings, such as whole class, individually, in pairs, and in small groups.

What all the *Investigations* units have in common is students working in depth on a number of problems, actively using mathematical tools, and consulting with peers as they find ways to solve the problems. The *Investigations* units allow significant time for students to think about the problems and to model, draw, write, and talk about their work. In addition to the investigations, the curriculum also includes classroom routines, ten-minute math activities, and games that provide practice and support to extend students' mathematical thinking.

- ***Every Day Mathematics*** (SRA. McGraw-Hill, Grades K-6)  
*Everyday Mathematics* is a Kindergarten through grade six curriculum intended to enrich the mathematical experiences of teachers and children. It builds on the fundamental mathematical strands identified below.
  - measures and measurement
  - reference frames
  - operations
  - patterns, functions and sequences
  - data and chance
  - geometry and spatial sense
  - numeration and order
  - algebra and the use of variables

Additionally, this curriculum supports teaching and learning by:

- Providing important concepts over consecutive years. Each grade level builds on and extends conceptual understandings established in prior grades.
  - Blending exposition and discussion, individual and group work, explorations, and projects. In each grade level of the program, students experience hands-on, student-centered, small-group activities that introduce them to various mathematics concepts.
  - Including cross-curricular projects that let children apply mathematics in other subject areas. In grades four and five, children participate in the year-long *World Tour* and *American Tour* projects, in which students apply mathematical concepts to learning about the United States and other countries.
- ***College Preparatory Mathematics*** (Grades 6-12)  
*College Preparatory Mathematics* (CPM) is a six-year secondary-school curriculum that integrates algebra and geometry content with conceptual understanding and problem solving skills. A complete curriculum includes mastering basic skills and procedures,

understanding mathematical principles, and acquiring problem solving strategies. The *CPM* program presents mathematical ideas in contexts that help students make sense of otherwise abstract principles. Students are taught how to gather and organize information about problems, break problems into smaller parts, and look for patterns that lead to solutions. Each course is built around several core ideas that are used to develop related topics, skills and procedures. Students master skills and come to the understanding of ideas over several days and weeks. Much of their classroom time is spent doing guided investigations, much like a math lab, that develop ideas in concrete, visual ways. They also apply their learning to realistic problems that require more than merely mimicking examples of rules.

## INTERVENTION PROGRAMS CURRENTLY UNDER REVIEW

- ***Math Recovery*** (Ohio State University, Grade 1)

*Math Recovery* focuses on first grade students who need an extra boost in their math skills in order to improve achievement in the classroom. Each first grade class is screened in basic math skills such as forward and backward counting, number recognition, sequencing numbers, and basic addition and subtraction facts. Those scoring the lowest in these areas receive a more in-depth assessment. From the assessments, children are selected to receive intense one-on-one instruction and small group instruction using a variety of strategies and manipulatives.

The *Math Recovery* program also extends these services into second grade as a follow-up to the first grade sessions. This is a tracking system to see if these students are maintaining success in the classroom.

- ***STAR Math*** (Grades 3-12)

*STAR Math* is a computer-adaptive math test and database that is an achievement-level learning information system that provides teachers with accurate math scores for students in grades three through twelve in fifteen minutes. It serves two primary purposes:

- Provides teachers with quick and accurate estimates of student's math skills,
- Offers sound estimates of students' math abilities relative to national norms.

*STAR Math* has been correlated with many norm-referenced examinations, which allows teachers to fine-tune instruction prior to the next regular testing cycle.

- ***MathLand*** (Creative Publications, Grades K-6)

*MathLand* is a comprehensive mathematics program for grades Kindergarten through six focusing on students' use of problem-solving approaches to investigate and understand mathematical content. The program is based on a 36-week school year and is divided into ten units at each grade level, with the exception of kindergarten, which has eight units. *MathLand* presents students with coherent, connected, and accessible mathematical experiences. The foremost learning goal for *MathLand* is that students become mathematically proficient and confident problem-solvers. Each unit of the program includes investigations, problems, and exercises that are interrelated. The materials emphasize development of skills such as problem solving, communication, reasoning, mathematical connections, estimation, measurement, statistics, probability, fractions, and decimals. Progressing through the years, students develop both basic skills and advanced conceptual understanding and problem solving skills through the use of *MathLand* materials.

*MathLand* is based on the NCTM standards. The program's goals are challenging, clear, and appropriate for the intended student population. The program was designed to involve students in hands-on investigations to foster problem solving, mathematical

communication, connections between disciplines and the real world, and reasoning skills. Concepts are studied in depth, allowing for skill development through exploration and activities, discussion, collaborative work, the use of hands-on materials, technology, graphical and pictorial displays, dramatizations, and oral and written presentations. The instructional design provides student involvement through open-ended activities for which the teacher becomes the coach, prompting student discussions. *MathLand* uses students' everyday life as the context for activities, beginning with students' everyday language and connecting it to mathematical language and symbols.

- ***College Preparatory Mathematics*** (Gr. 6-12)  
*College Preparatory Mathematics (CPM)* is a six-year secondary-school curriculum that integrates algebra and geometry content with conceptual understanding and problem solving skills. A complete curriculum includes mastering basic skills and procedures, understanding mathematical principles, and acquiring problem solving strategies. The *CPM* program presents mathematical ideas in contexts that help students make sense of otherwise abstract principles. Students are taught how to gather and organize information about problems, break problems into smaller parts, and look for patterns that lead to solutions. Each course is built around several core ideas that are used to develop related topics, skills, and procedures. Students master skills and come to understand ideas over several days and weeks. Much of their classroom time is spent doing guided investigations, much like a math lab, that develop ideas in concrete, visual ways. They also apply their learning to realistic problems that require more than merely mimicking examples of rules.
  
- ***Moving with Mathematics*** (Grades K-8)  
*Moving with Math* is a complete program providing comprehensive instruction on all essential skills. The program adheres to NCTM curriculum guidelines.
  - Diagnostic Pre-Tests identify students' appropriate level and specific unmastered objectives. The system is easy to use because the number of each question on the Pre-Test matches the number of the tested objective. Test results are recorded on the individual and class record sheets. The number-coded objectives make it easy to prescribe appropriate pages from *Moving with Mathematics* or *Skill Builders* to teach unmastered objectives.
  - Student Books and *Skill Builders* activity sheets and games enhance instruction.
  - Post-Tests measure progress in mastering objectives and help assess immediate results. Re-teaching can be prescribed from *Skill Builders*.
  - Maintenance tests are the key to long-term retention. Daily reviews in *Math Capsules* help students retain essential math skills and improve achievement on standardized tests. The number of times each objective is reviewed varies according to the difficulty of the objective in classroom testing.

## **MATHEMATICS ENRICHMENT COMPONENTS CURRENTLY UNDER REVIEW**

According to the consensus of the best practitioners and top researchers, an effective mathematics program should be integrated with a strong enrichment program. Research strongly indicates that good enrichment programs enhance, motivate, and support student achievement and allow students to perform the activities fluently and with flexibility. Enrichment programs in Tulsa Public Schools will be well integrated with excellent classroom instruction.

- ***Accelerated Math*** (Grades K-12)

*Accelerated Math* is a computerized math management program designed to:

- Motivate and monitor daily practice of foundational skills,
- Help every student master every objective,
- Generate personalized algorithm-based math assignments—each assignment being unique and promoting cooperative learning,
- Reduce paperwork (system scores student daily work and tests),
- Automatically identify areas requiring immediate intervention,
- Enhance existing curriculum materials,
- Improve student success in mathematics.

- ***Hands-On Equations*** (Borenson & Associates, Grades 3-9)

*Hands-On Equations* is:

- A patented teaching methodology empowering teachers to successfully present essential algebraic concepts to students in grades three to high school,
- A program which provides students with an intuitive understanding of the most fundamental algebraic concepts,
- A system that makes algebraic linear equations accessible to third grade gifted students, fourth grade average students, and fifth grade learning-disabled students,
- A valuable, hands-on approach to use in a traditional pre-algebra class,
- A program which may be used at the high school or college level, or in adult education, with students who have not previously experienced success with algebraic concepts,
- A program which does not require any algebraic prerequisites,
- A system which provides students with a strong foundation for later algebraic studies.

- ***Kumon Math*** (Grades PreK-Calculus)

With the *Kumon* method of learning, students can develop a solid foundation in math as well as improved concentration and enhanced self-confidence. The key to learning with *Kumon* is to start the students at a comfortable starting point, where they can consistently solve 100% of the problems in a given amount of time. This approach increases self-confidence and allows the students to experience success right from the start. The

*Kumon* students advance according to individual ability rather than age or grade level. To accomplish this, the *Kumon* curriculum presents math concepts in small increments, providing ample practice for each step. This method of individualized learning encourages independent learning, giving students the knowledge and skills to study on their own. Through repeated practice, students develop mastery of skills before moving on to a higher level of study. Since the goal of the *Kumon* method is to make high school study easy, *Kumon* students are encouraged to reach an advanced level of study.

## **IMPROVING MATHEMATICS THROUGH PROFESSIONAL DEVELOPMENT**

The goal of all children computing on or above grade level by the end of third grade can only be accomplished with powerful, systemic and sustained professional development.

“Changing instructional practice is a slow business. Teaching is a complex activity.... Change follows after teachers acquire new knowledge and experiment with using that knowledge in their own classrooms” (Allington & Cunningham, 1996).

The Educational Research Service has identified common characteristics of schools with large numbers of low-income children that are achieving at a higher level than is expected. These schools are organized so that teachers can consider how what they are doing affects student achievement and look at alternatives. The teachers who do the work are entrusted with the responsibility to improve achievement. These schools focus on standards and clear goals and the principals work daily to keep the focus. Teachers believe and are committed to ensuring that all students can reach high standards. Teachers work in teams over a period of time on making results-oriented, data-driven changes to improve student achievement (Cawelti, 1999).

Our plan is as follows.

- Each site will designate a lead mathematics teacher or department chair who will work with both site and district personnel to improve math achievement.
- Mathematics Resource Teachers/Coaches will support each school.
- The district will provide training based on the train the trainer model for the lead mathematics teachers and department chairs.
- Lead mathematics teachers and department chairs will provide school-based training and, along with district personnel, facilitate the work of the learning communities at each school.

The content for professional development refers to the actual beliefs, skills, and knowledge that effective educators must possess or acquire. The content will be carefully chosen to enable teachers and administrators to:

- understand how to provide school environments and instruction that are responsive to the developmental needs of their students,
- develop and implement school and classroom-based management which maximizes student learning,
- acquire the knowledge, skills, and behaviors needed to ensure that equitable and quality mathematics instruction is provided to all students,
- provide challenging, developmentally-appropriate mathematics curricula that engage students in integrative ways of thinking and learning,

- use research-based teaching strategies appropriate to their mathematics objectives and their students,
- demonstrate high expectations for student learning,
- facilitate staff collaboration with and support of families for improving student performance in mathematics,
- use various types of performance assessments.

*National Staff Development Council's Standards for Staff Development (1994)*

Professional development in mathematics to be provided by Tulsa Public Schools will include, but not be limited to, the following.

- Assessment for diagnosis, prescription and instruction
- Standardized test score analysis
- Early skill development
- Flexible grouping for instruction
- Meeting the needs of subgroups such as English Language Learners, Special Needs, and Gifted/Talented
- Individualizing instruction
- Strategies for integrating mathematics with other content areas
- Teacher peer coaching
- Parental involvement
- Strategies for successfully working with students of poverty
- Collaboration between teachers and library media specialists
- Components and frameworks of successful mathematics programs
- Investigation of scientifically research-based national models
- Use of a mathematics rubric for reflection (see Appendix).
- Use of a Principals' Mathematics Checklist (see Appendix).

## **IMPROVING MATHEMATICS THROUGH LIBRARY MEDIA PROGRAMS**

A growing body of research demonstrates that library media programs play a critical role in supporting student achievement. Therefore, students will work in groups, or individually, to produce a product (a written or oral report, project, map, poster, play, etc.) that demonstrates learning using a variety of resources to answer a question or solve a problem using *The Organized Investigator Model*.

### ***The Organized Investigator* Research Model**

#### **Questions and Wonders**

#### **Establish a focus for investigation.**

Teacher/Library Media Specialist Steps:

- Based on content objectives, select a unit of study,
- Collaborate with librarian as needed in unit planning,
- Help students identify their interests within the topic,
- Access students' prior knowledge,
- Help students formulate a relevant/essential question for investigation,
- Decide upon a range of end products,
- Determine if students have an option of communication/presentation formats,
- Help students manage the project by breaking it into steps with separate due dates.

#### **Reminders**

- Schedule a teacher/librarian planning meeting.
- Collaborate with your library media specialist to identify potential appropriate resources and to plan integration of necessary research skills.
- Provide students with background information on the topic through books, centers, posters, videos, experiences, speakers, field trips, etc.
- Use a *KWL* chart or other graphic organizer to assess student prior knowledge.
- Model brainstorming and webbing to focus the topic and formulate research questions.
- Develop rubrics and decide due dates and tentative timeline.
- Break project into steps.

## **Finds and Sorts**

### **Identify and collect helpful information.**

Teacher and librarian plan jointly to:

- Introduce (or review) a variety of resources, including print, electronic and human, stressing their characteristics/differences,
- Teach the purpose and use of indexes, table of contents, cross-references, and electronic resources,
- Discuss and help select resources for appropriateness, e.g., reading level, copyright, viewpoint, and format,
- Review shortcut strategies such as skimming and scanning, reading captions, subject headings, etc.,
- Help identify key words relevant to their question,
- Provide adequate time for instruction and for the search.

### **Reminders**

- Agree upon who (librarian and/or teacher) teaches each part of the research process.
- Dialog possible sources with students.
- Provide ongoing, “just in time” learning for specific research sources.

## **Consumes and Gulps**

### **Gather, read, and view information.**

Teacher/Library Media Specialist Steps:

- Provide students a format for taking notes and recording bibliographic data,
- Provide time for research/reading.

### **Reminders**

- Discuss format for notes including use of databases, computer files, spreadsheets or note cards.
- Monitor student progress and their selection of resources.
- Check student work, helping to refine the question and choice of keywords.
- Redirect focus as needed.
- Suggest alternative sources as needed.

## **Thinks and Creates**

Teacher/Library Media Specialist Steps:

- Help students reflect and interpret the information they have found,
- Help students assess the information,
- Help students organize the information.

### **Reminders**

- Help students relate their findings to the essential question.
- Teach organizational techniques such as,
  - Color coding information,
  - Outlining,
  - Using note cards,
  - *Organized Investigator Model* strategies,
  - Use of databases and spreadsheets.
- Help students compare and contrast information.
- Teach the following assessment skills, as needed:
  - Evaluating evidence
  - Recognizing distortion and over simplification
  - Distinguishing between fact and opinion
  - Recognizing bias/source's perspective
  - Recognizing irrelevance
  - Recognizing expert sources
  - Distinguishing primary and secondary sources.

## **Summarizes and Concludes**

Teacher/Library Media Specialist Steps:

- Discuss students' findings by helping them to:
  - Examine evidence,
  - Draw conclusions,
  - Paraphrase information,
  - Summarize findings.

### **Reminders**

- Make sure that student conclusions are based on research rather than subjective opinion.
- Make sure students are objective in their conclusions.

## **Communicates**

Teacher/Library Media Specialist Steps:

- Review project choices, guidelines, and rubrics,
- Guide students in selecting the appropriate format for communication,

- Assist students in creating a bibliography,
- Teach students how to use the technology required for the project,
- Allow time for product development,
- Provide time and format for peer review,
- Allow time for revision and editing,
- Schedule and allow time for presentation.

### **Reflects on Process and Product**

Teacher/Library Media Specialist Steps:

- Evaluate projects against rubric,
- Provide feedback to students,
- Discuss successful and unsuccessful search strategies with students.

## **PARENT INVOLVEMENT**

The total community, including parents of TPS students, must work together to ensure that every child in Tulsa Public Schools is proficient in mathematical skills and problem solving, as appropriate for his/her grade level, by the end of third grade. Teachers, administrators, and library media specialists will focus mathematics instructional strategies on the established standards and benchmarks for the district. Parents, mentors, volunteers, Adopt-a-School partners, and Partners-in-Education are needed to work with the schools to support our students as they strive to achieve in mathematics. Following is a guide for parental involvement that, if used, will assist students in developing their mathematical knowledge and competencies.

### **PARENTAL INVOLVEMENT**

#### ***Early Childhood: Where Learning Begins - Mathematics - June 1999***

The National Council of Teachers of Mathematics (NCTM), the world's largest organization devoted to improving mathematics education, has developed a set of mathematics concepts, or standards that are important for teaching and learning mathematics. There are two categories of standards: thinking math standards and content math standards. The thinking standards focus on the nature of mathematical reasoning, while the content standards are specific math topics. Each of these activities touches one or more content areas and may touch all four thinking math areas.

The four thinking math standards are problem solving, communication, reasoning, and connections. The content math standards are patterns and relationships, number sense and numeration, geometry and spatial sense, measurement, fractions, estimation, and statistics and probability. Below is a description of each standard as well as strategies that parents can use to create activities that build skills in each of these areas.

#### **Thinking Mathematics**

**1. Problem solving** is key in being able to do all other aspects of mathematics. Through problem solving children learn that there are many different ways to solve a problem and that more than one answer is possible. It involves the ability to explore, think through an issue, and reason logically to solve routine as well as non-routine problems. In addition to helping with mathematical thinking, this activity builds language and social skills, such as working together.

#### **What Parents Can Do**

- Children are naturally curious about everyday problems. Invite children to figure out solutions to everyday situations. Talking about the problem, asking children for ways to solve it, and then asking how they came up with those solutions can do this.

- Encourage children to suggest problems and ask questions, too. Children will learn how to figure things out and will learn that many problems can be solved in several different ways.

**2. Communication** means talking with children and listening to them. It means finding ways to express ideas with words, diagrams, pictures, and symbols. When children talk, it helps them think about what they are doing and makes their own thoughts clearer. As a bonus, talking with children improves their vocabulary and helps develop literacy and early reading skills as well.

#### **What Parents Can Do**

- Talk with children and listen to what they have to say.
- Read children's books that rhyme, repeat, or have numbers in them (available at your school or local library). This is a great way to communicate using mathematics. All communication does not have to be in words. Mathematics can be represented in ways other than talking. Children can make diagrams or draw pictures to solve problems or represent numbers. They can use concrete objects like pieces of paper or even fingers to represent numbers.

**3. Reasoning** is used to think through a question and come up with a useful answer. It is a major part of problem solving.

#### **What Parents Can Do**

- To promote reasoning, ask children questions and give them time to think about the answer. By simply asking questions and listening to answers, parents can help children learn to reason.
- Ask children to figure out why something is the way it is and then check out their ideas. Let them think for themselves, rather than try to figure out what answer you want to hear.

**4. Connections** are drawn to other subjects. Mathematics is not isolated skills and procedures. Mathematics is everywhere and most of what we see is a combination of different concepts. A lot of mathematics relates to other subjects like science, art, and music. Most importantly, math relates to things we do in the real world every day. Connections make mathematics easier for children to understand because they allow children to apply common rules to many different things.

#### **What Parents Can Do**

- Ask children to think about and solve problems that arise in your everyday activities.

For example, ask children to help put the groceries away. They will practice sorting (the cereal boxes and the soup cans) and experiment with relative size and shape, and see how the big boxes take up more room than the smaller ones.

- Look for mathematics in everyday life and don't worry about what the particular aspect of mathematics might be. Something as simple as pouring water into different sized cups and thinking about which cup will hold more is a low-key activity that actually involves estimation, measurement, and spatial sense.

## **Content Mathematics**

**1. Patterns and relationships:** Patterns are things that repeat; relationships are things that are connected by some kind of reason. They are important because they help develop understanding of the underlying structure of things; they help add a feeling of confidence and capability of knowing what will come next, even when we can't see it yet. Patterns and relationships are found in music, art, and clothing, as well as in other aspects of mathematics such as counting and geometry. Understanding patterns and relationships means understanding rhythm and repetition as well as ordering from shortest to longest, smallest to largest, sorting, and categorizing.

### **What Parents Can Do**

- Help children find patterns in designs and pictures, as well as in movement and in recurring events such as the days of the week or the seasons of the year.
- For a hands-on activity try stringing wooden blocks or pasta necklaces into a simple red-blue-red-blue pattern. As children get older they can reproduce and create more complex patterns.

**2. Number sense and numeration:** Number sense is much more than merely counting, it involves the ability to think and work with numbers easily and to understand their uses and relationships. Number sense is about understanding the different uses for numbers (describe quantities and relationships, informational tools). Number sense is the ability to count accurately and competently, to be able to continue counting, or count on, from a specific number as well as to count backwards, to see relationships between numbers, and to be able to take a specific number apart and put it back together again. It is about counting, adding, and subtracting. Counting and becoming familiar with numbers will help your children understand all other aspects of math.

### **What Parents Can Do**

- Count anything and everything! Count real things to help children use their own experience with objects to better understand numbers. Therefore, one of the best math activities you can do with your children is to have them count real objects.

- To help children learn to count accurately and efficiently, up and down:
  - Point out that counting lets them know how many things there are in a group
  - Point to the object as you recite each number name
  - Use fingers to count. Put up a finger one at a time as you count it: fingers are tools you always have with you
  - Help your children count without skipping numbers or counting something twice.
  
- To help children learn that numbers are used to describe quantities and relationships, encourage them to:
  - Sort objects looking for similarities in either color, shape, or size
  - Sort objects looking for differences, such as which box is bigger.
  
- Talk to children about what numbers are used for, such as keeping score in a game or finding an apartment or street address.

**3. Geometry and spatial sense:** Geometry is the area of mathematics that involves shape, size, space, position, direction, and movement, and describes and classifies the physical world in which we live. Young children can learn about angles, shapes, and solids by looking at the physical world. Spatial sense gives children an awareness of themselves in relation to the people and objects around them.

### **What Parents Can Do**

- Parents can help children learn geometry by helping them learn to identify different angles, shapes, and three-dimensional figures. Ask them to identify and describe different shapes, to draw them in the air with their finger, to trace over them with their fingers, and to draw them on paper.
  
- Children learn geometry best through hands-on experiences. Things they can touch and manipulate are particularly helpful, like blocks, boxes or containers, shape sorters, and puzzles. Even cutting the sandwiches they eat into different shapes and letting them fit them together or rearrange them helps children learn about geometry.
  
- Geometry and spatial sense help children with directions and finding their way around. Let them climb in and out of boxes, on or around furniture, going under, over, around, through, into, on top of, and out of different things to experience themselves in space.

**4. Measurement:** Measurement is finding the length, height, and weight of an object using units like inches, feet, and pounds. Time is measured using hours, seconds, and minutes.

Measurement is an important way for young children to look for relationships in the real world. By practicing measurement children will learn how big or little things are and how to figure that out.

### **What Parents Can Do**

- Standard measurements like inches, feet, and yards are only one way to measure. Let children pick their own unit for measurement, for example: "Raymond is five cereal boxes tall."
- Many daily activities involve measurement: cooking, gardening, grocery shopping, sewing, and woodworking are only a few examples. Keep children involved in these chores by talking with them about what is being done, and by asking them to help you when possible.
- To help young children understand the concept of time, you may want to do the following.
  - Teach children that some activities take longer than others. Compare one activity with another to figure out what takes more time. Start by asking simple questions like "Who can stand on one foot longer?"
  - Set end of time limits. "You can only play for 5 more minutes, then we have to go catch the bus." At first children won't know what the minutes mean, but gradually the children will understand the idea of time passing.
  - Relate units of time to counting by using a watch to time events and counting the ticks on the second hand of the watch by saying, "1 second, 2 seconds, 3 seconds" without timing the event. This uses counting to help children develop a sense of the passing of time.
  - Start time talk with ideas like "after lunch" or "after dinner" that provide solid milestones for children. It is not until children are older that they understand more abstract notions like yesterday, today, and tomorrow.

**5. Fractions:** Fractions represent parts of a whole. A very young child will see something cut into three pieces and will believe that there is more after cutting it than before it was cut. This is typical and should not cause alarm in parents. It is one example of how children and adults think differently!

To understand fractions, children need to think about:

- what the whole unit is,
- how many pieces are in the unit,
- if the pieces are the same size.

## What Parents Can Do

- Many sharing activities help children understand fractions by dividing food, chores, or treats into equal portions. Cutting up pizzas or sharing a pan of brownies are examples of ways to get children thinking about fractions.

**6. Estimation:** To estimate is to make an educated guess as to the amount or size of something. To estimate accurately, numbers and size have to have meaning. Very young children will not be able to estimate accurately, because they are still learning these concepts. They first need to understand concepts like more, less, bigger, and smaller. When children use estimation, they learn to make appropriate predictions, to obtain reasonable results, and they learn math vocabulary such as "about," "more than," and "less than."

It is important for children to learn:

- how to use estimation,
- when the technique is appropriate,
- when the solution is reasonable.

## What Parents Can Do

- Regularly use words such as "about," "near," "approximately," "in between," "around," "more than," and "fewer than."
- To practice estimation, use any daily activity such as eating or going to the store. Ask children to make an estimate of how much, how long, or how many. When you find the actual answer, compare it with the estimate you originally made. This will help children learn to make reasonable guesses.
- Estimation activities may sound complicated, but they don't have to be. Just take a guess about something, like which one of his/her friends is the tallest, and then check it out for accuracy.
- To help older children learn estimation, write down the estimate and then the actual count. If you repeat the problem or give a similar problem, children will eventually estimate closer to the real count. It isn't important to get the "right" answer, but to see how close children can come. Help them practice the technique to improve their estimation skills.

**7. Statistics and probability:** Using graphs and charts, people organize and interpret information and see relationships. Graphing is another way to show and see information mathematically. Charts, including calendars, can be used to organize everyone's weekly activities. Even older children in elementary school may find it hard to keep track of calendars, but, when adults use them with children, calendars can be helpful tools to learning

and understanding how we organize information.

Statistics, like batting averages in baseball, tell stories about our world. We know which player is having the best season and which batter is most likely to hit a home run. Probability tells the likelihood of something occurring.

### **What Parents Can Do**

Practically everything you do is "chartable." For example, you can take the stickers from bananas, apples, or pears and place them in columns on a piece of paper. At the end of the week you can count them up to see how many of each type of fruit you ate. Graphs help some children reach a greater understanding of numbers because children can see quantities displayed on paper. This may help them understand math more than looking at numbers on paper.

- Use color forms or stickers to record any regular daily activity. Put a color form next to the phone each time it rings, or have people put one next to the front door to record the number of times someone enters.

## CONCLUSION

The message is clear. District, state, and national tests indicate a serious decline in mathematics scores. Children who do not understand mathematical concepts will have difficulty being successful in school or in life. Mathematics must be a priority for this school district.

A comprehensive mathematics initiative is needed to ensure that all students will receive high quality, mathematics instruction so that they may become competent problem solvers. Effective instruction that prepares students for life in the twenty-first century includes the development of higher order thinking skills along with the development of a deeper understanding of mathematical concepts. Less stress should be placed on rote memorization, lectures, and drill and practice. This can be accomplished through the increased use of manipulatives or other concrete materials and through teacher encouragement of student discussion and explanation of mathematical ideas and processes. Problem solving strategies and techniques must also receive emphasis at every grade level.

Due to changes in technology, there should be greater use of calculators and computers in the mathematics classroom. To ensure accuracy when using calculators and computers, estimation and mental arithmetic must be taught thoroughly.

In creating an environment conducive to mathematical learning, the role of the teacher will be to guide student thinking, to collaborate with students in the discussion of ideas, to create developmentally appropriate environments and materials, to ask probing questions, to extend or clarify concepts, and to listen to and encourage individual thinking.

The Tulsa Model for School Improvement is the overall framework for improving student achievement. ***Tulsa COUNTS!*** defines specific strategies and approaches that will improve student achievement in mathematics. Implementing The Tulsa Model for School Improvement, including ***Tulsa COUNTS!***, will provide the opportunity for all students to receive high quality mathematics learning experiences every day.

# APPENDIX

## QUESTIONING STRATEGIES

### QUESTIONING FOR QUALITY THINKING

Recalling:

Who, what, when, where, how \_\_\_\_\_?

Comparing:

How is \_\_\_\_\_ similar to/different from \_\_\_\_\_?

Identifying Attributes and Components:

What are the characteristics/parts of \_\_\_\_\_?

Classifying:

How might we organize \_\_\_\_\_ into categories?

Ordering:

Arrange \_\_\_\_\_ into sequence according to \_\_\_\_\_.

Identifying Relationships and Patterns:

Develop an outline/diagram/web of \_\_\_\_\_.

Representing:

In what other ways might we show/illustrate \_\_\_\_\_?

Identifying Main Ideas:

What is the key concept/issue in \_\_\_\_\_?

Retell the main idea of \_\_\_\_\_ in your own words.

Identifying Errors:

What is wrong with \_\_\_\_\_?

Inferring:

What might we infer from \_\_\_\_\_?

What conclusions might be drawn from \_\_\_\_\_?

Predicting:

What might happen if \_\_\_\_\_?

Elaborating:

What ideas/details can you add to \_\_\_\_\_?

Give an example of \_\_\_\_\_.

Summarizing:

Can you summarize \_\_\_\_\_?

Establishing Criteria:

What criteria would you use to judge/evaluate \_\_\_\_\_?

Verifying:

What evidence supports \_\_\_\_\_?

How might we prove/confirm \_\_\_\_\_?

### STRATEGIES TO EXTEND THINKING

- Remember wait time:  
Provide at least five seconds of thinking time after a question and after a response.
- Ask follow-ups:  
e.g., “Why? How do you know? Do you agree? Will you give an example? Can you tell me more?”
- Cue responses to open ended questions:  
e.g., “There is not a single correct answer to this question. I want you to consider alternatives.”
- Use think-pair-share:  
Allow individual thinking time, discussion with a partner, and then open up for class discussion.
- Call on students randomly:  
Avoid the pattern of only calling on those students with raised hands.
- Ask students to unpack their thinking:  
e.g., “Describe how you arrived at your answer.”
- Ask for summary to promote active listening:  
e.g., “Could you please summarize our discussion thus far?”
- Challenge reasoning:  
Require students to defend their reasoning against different points of view.
- Survey the class:  
e.g., “How many people agree with the author’s point of view?” (thumbs up, thumbs down)
- Allow for student calling:  
e.g., “Richard, will you please call on someone to respond?”
- Encourage student questioning:  
Provide opportunities for students to generate their own questions.

**Mathematics Rubric for Elementary Teachers**  
*A Tool for Reflection*

Directions: Please check the behaviors according to frequency of use.

	Seldom	Sometimes	Always
1. I act as a facilitator and coach.			
2. I provide “real life” problems every day to employ the application of knowledge and skills.			
3. I provide daily problem solving opportunities and require the students to justify the process and their answers.			
4. I use a variety of methods of instruction and classroom stimulation—projects, manipulatives, flexible grouping, field trips, speakers, and technology.			
5. I provide students opportunities to actively participate intellectually, physically and emotionally.			
6. I encourage critical thinking skills, rather than rote memorization.			
7. I model and communicate with students that they are valued.			
8. I communicate and demonstrate that there are multiple solutions rather than a single method to achieve the answers.			
9. I provide a safe and secure, student-centered environment and encourage students to take risks.			
10. I provide dialogue between teacher and student, and encourage self-reflection and self-evaluation.			
11. I ask high-level questions that engage students in higher level thinking skills—data analysis, synthesis of results and evaluation of potential solutions.			
12. I provide lessons addressing the multiple intelligences and different learning styles.			
13. I provide a meaningful mathematics curriculum aligned with the TPS pacing calendar and the strategies of the Tulsa Model.			
14. In addition to the required testing, I use a variety of ongoing assessments embedded in instruction to determine a student’s progress --observations, portfolio assessment, electronic portfolios, interview assessments, logs and journals, and projects.			
15. I communicate the expectations and assist students in setting personal goals for continued learning.			
16. I provide intervention strategies for students who are not meeting the standards.			
17. I collaborate with the media specialist to integrate technology to enrich the students’ learning of mathematics.			
18. I encourage parental involvement in student learning			

**Mathematics Rubric for Secondary Teachers**  
*A Tool for Reflection*

Directions: Check the behaviors according to frequency of use.

	Seldom	Sometimes	Always
1. I use authentic projects to help integrate students' conceptual knowledge with their procedural knowledge			
2. I use a variety of comprehension strategies, such as graphic organizers, (KWL charts, t-charts, Venn diagrams), note taking and summarizing; and questioning.			
3. I use a variety of activities to help develop understanding of key concepts.			
4. I use tangible objects to construct the meaning of complex mathematical concepts.			
5. I use a variety of ongoing assessments to determine a student's progress			
6. I provide intervention strategies for students who are not meeting standards.			
7. I provide periodic individual conferences for feedback and instruction.			
8. I use flexible groups to allow students to articulate their own understanding of the content and to teach each other.			
9. I give students opportunities to discuss what they have learned.			
10. I use a variety of ongoing assessments embedded in instruction to determine a student's progress, i.e., observations, portfolio assessment, electronic portfolios, interview assessments, logs and journals, and projects.			
11. I provide electronic resources for students to integrate, apply, and extend themes, concepts, and skills.			
12. I ask high-level questions that cause students to analyze, compare, contrast, and interpret.			
13. I collaborate with the library media specialist to integrate mathematics and other content areas to facilitate use of the <i>Organized Investigator</i> research model.			

## Mathematics Checklist for Elementary Principals

Teacher \_\_\_\_\_ Grade \_\_\_\_\_ Date \_\_\_\_\_

### Teacher Behaviors:

- \_\_\_ Acts as a facilitator and coach.
- \_\_\_ Provides “real life” problems every day to employ the application of knowledge and skills.
- \_\_\_ Provides daily problem solving opportunities and requires the students to justify the process and their answers.
- \_\_\_ Uses a variety of methods of instruction and classroom stimulation, e.g., projects, manipulatives, flexible grouping, field trips, speakers, and technology.
- \_\_\_ Provides students opportunities to actively participate intellectually, physically, and emotionally.
- \_\_\_ Encourages critical thinking skills, rather than rote memorization.
- \_\_\_ Models and communicates with students that they are valued.
- \_\_\_ Communicates and demonstrates that there are multiple solutions rather than a single method to achieve the answers.
- \_\_\_ Provides a safe and secure, student-centered environment and encourages students to take risks.
- \_\_\_ Provides dialogue between teacher and student, and encourages self-reflection and self-evaluation.
- \_\_\_ Asks high-level questions that engage students in higher level thinking skills, e.g., data analysis, synthesis of results, and evaluation of potential solutions.
- \_\_\_ Provides lessons addressing the multiple intelligences and different learning styles.
- \_\_\_ Provides a meaningful mathematics curriculum aligned with the TPS pacing calendar and the strategies of the Tulsa Model.
- \_\_\_ Uses a variety of ongoing assessments embedded in instruction to determine a student’s progress, e.g., observations, portfolio assessment, electronic portfolios, interview assessments, logs, journals, and projects, in addition to the required testing.
- \_\_\_ Communicates the expectations and assists students in setting personal goals for continued learning.
- \_\_\_ Provides intervention strategies for students who are not meeting the standards.
- \_\_\_ Collaborates with the media specialist to integrate technology to enrich the students’ learning of mathematics.
- \_\_\_ Encourages parental involvement in student learning.

### Site Profile for Improvement of Mathematics Elementary Schools

**2003-2004**

**Note: This is the planning tool you will use as you develop your math goal for the Site Improvement Plan.**

A. Identify the assessments used at your site:

Informal

- Observation
- Portfolios
- End-of-unit Tests
- Projects
- Journals

Formal

- Standardized Test \_\_\_\_\_
- Benchmark Test
- Textbook Selection Test
- S.T.A.R.
- Other

B. Analyze the standardized assessment data. Determine the annual improvement goal to progress from the 2002 baseline to the goal of 90% of third grade students who will be proficient in mathematics by May 1, 2007.

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C. Identify intervention strategies for students who are not meeting the standards.

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D. Based on the results of the standardized tests, describe the professional development plans for the 2003-2004 school year.

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E. List strategies for teachers to interconnect mathematics among the disciplines.

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F. Describe methodologies to engage students in real-life problem-based learning for each

content standard. Include the products or projects that demonstrated the students' proficiency. (See Organized Investigator Model.)

Patterns: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Number Sense: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Number Operations and Computation: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Geometry and Measurement: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Data Analysis: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Committee members having input should include an administrator, teachers representing all content areas, including media specialists, and parents.

Name	Position/Title
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

School \_\_\_\_\_ Date submitted \_\_\_\_\_

## Mathematics Checklist for Secondary Principals

Teacher \_\_\_\_\_

Course \_\_\_\_\_

Date \_\_\_\_\_

### Suggested teacher behaviors:

- \_\_\_\_\_ Uses authentic projects to help integrate students' conceptual knowledge with their procedural knowledge.
- \_\_\_\_\_ Uses a variety of self-monitoring comprehension strategies such as graphic organizers (time lines, KWL charts, t-charts, Venn diagrams), note taking and summarizing, and questioning.
- \_\_\_\_\_ Uses a variety of activities to help develop understanding of key concepts.
- \_\_\_\_\_ Uses tangible objects to construct the meaning of complex mathematical concepts.
- \_\_\_\_\_ Uses a variety of ongoing assessments to determine a student's progress.
- \_\_\_\_\_ Provides intervention strategies for students who are not progressing.
- \_\_\_\_\_ Provides periodic individual conferences for feedback and instruction.
- \_\_\_\_\_ Uses flexible groups to allow students to articulate their own understanding of the content and to learn from each other.
- \_\_\_\_\_ Gives students opportunities to discuss what they have learned.
- \_\_\_\_\_ Uses data from various tests to help drive instruction.
- \_\_\_\_\_ Provides (identifies and demonstrates use of) electronic resources for students to integrate, apply, and extend themes, concepts, and skills.
- \_\_\_\_\_ Asks high-level questions that cause students to analyze, compare, contrast, and interpret.
- \_\_\_\_\_ Collaborates with the library media specialist to integrate mathematics and other content areas to facilitate use of the Organized Investigator research model.

**Site Profile for Improvement of Mathematics  
Secondary Schools  
2003 – 2004**

**Note: This is the planning tool you will use as you develop your math goal for the site Improvement Plan.**

Analyze the standardized assessment data especially in the area of mathematics computation, problem solving, and reasoning:

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Determine annual improvement goals (see API):

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Based upon the results of standardized tests in the area of mathematics problem solving, reasoning, and computation, and the annual improvement goals, describe professional development needs for the 2003 – 2004 school year:

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Plans for faculty and/or team meetings devoted to the implementation of instructional strategies that support mathematics in other content areas include:

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Discuss plans for students to be involved in information investigation. Students may work in groups or individually to produce a product (a written or oral report, project,

map, poster, play, etc.) that demonstrates learning using a variety of resources to answer a question or solve a problem: (see Organized Investigator Model)

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Committee members having input should include an administrator, teachers representing all content areas including library media specialists, and parents:

Name	Position/Title

School \_\_\_\_\_ Date submitted \_\_\_\_\_

***Tulsa COUNTS!***  
**Time Line for Implementation**

**Fall 2003**

- Designate site mathematics leader/department chair at each site.
- Establish site teams:
  - Grade level
  - Multi-level
    - Begin meeting to engage in action research to study student test results in mathematics.
    - Report findings to Principal and other staff members.
    - Set goal(s) for the year.
    - Begin study of *ED Thoughts – What We Know About Mathematics Teaching and Learning*.
    - Choose two critical skills areas to target.
    - Identify means to accomplish goals.
- Develop math profile.
- Identify and participate in Professional Development activities.
- Begin study and selection of standards-based curriculum.

**Winter 2004**

- Attend the textbook caravan.
- Plan 2004-2005 professional development activities.
- Provide training in *Connect to the Standards* for PreK-5 lead teachers and 6-8 department chairs.
- Continue study of *ED Thoughts – What We Know About Mathematics Teaching and Learning*, McREL.

**Spring 2004**

- Continue training in *Connect to the Standards and ED Thoughts*.
- Evaluate math profile, modify and adjust for year 2004-2005.
- Vote on district textbook series.
- Schedule Professional Development training for new mathematics adoption.

**Tulsa Public Schools Mathematics Testing Requirements  
2003-2004**

Grade 1	Benchmark Tests
Grade 2	Benchmark Tests
Grade 3	Benchmark Tests and Stanford 9 Test
Grade 4	Benchmark Tests
Grade 5	Benchmark Tests and OK Core Curriculum Test
Grade 6	Benchmark Tests
Grade 7	Benchmark Tests
Grade 8	Benchmark Tests and OK Core Curriculum Test
Grade 9	Benchmark Tests
Grade 10	Benchmark Tests
EOI	Algebra I

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