TULSA PUBLIC SCHOOLS

High School
Algebra I & II and Geometry

(5/4-15/20)

Distance Learning Activities

Dear families,

These learning packets are filled with grade level activities to keep students engaged in learning at home. We are following the learning routines with language of instruction that students would be engaged in within the classroom setting. We have an amazing diverse language community with over 65 different languages represented across our students and families.

If you need assistance in understanding the learning activities or instructions, we recommend using these phone and computer apps listed below.

Google Translate

- Free language translation app for Android and iPhone
- Supports text translations in 103 languages and speech translation (or conversation translations) in 32 languages
- Capable of doing camera translation in 38 languages and photo/image translations in 50 languages
- Performs translations across apps

Microsoft Translator

- Free language translation app for iPhone and Android
- Supports text translations in 64 languages and speech translation in 21 languages
- Supports camera and image translation
- Allows translation sharing between apps

Queridas familias:

Estos paquetes de aprendizaje tienen actividades a nivel de grado para mantener a los estudiantes comprometidos con la educación en casa. Estamos siguiendo las rutinas de aprendizaje con las palabras que se utilizan en el salón de clases. Tenemos una increíble y diversa comunidad de idiomas con más de 65 idiomas diferentes representados en nuestros estudiantes y familias.

Si necesita ayuda para entender las actividades o instrucciones de aprendizaje, le recomendamos que utilice estas aplicaciones de teléfono y computadora que se enlistan a continuación:

Google Translate

- Aplicación de traducción de idiomas para Android y iPhone (gratis)
- Traducciones de texto en 103 idiomas y traducción de voz (o traducciones de conversación) en 32 idiomas
- Traducción a través de cámara en 38 idiomas y traducciones de fotos / imágenes en 50 idiomas
- Realiza traducciones entre aplicaciones

Microsoft Translator

- Aplicación de traducción para iPhone y Android (gratis)
- Traducciones de texto en 64 idiomas y traducción de voz en 21 idiomas
- Traducción a través de la cámara y traducción de imágenes
- Permite compartir la traducción entre aplicaciones

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If you need help, please leave a message at 918-746-7500 and an enrollment specialist will return your call or email enroll@tulsaschools.org.

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TULSA PUBLIC SCHOOLS

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1 AL 21 DE MAYO DE 2020

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Si necesita ayuda, deje un mensaje al 918-746-7500 y un especialista en inscripción le devolverá la llamada. También puede enviarnos un correo electrónico a enroll@tulsaschools.org.

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Lesson 11 Cumulative Practice Problems Review – Use videos from previous lessons to help.

1. Draw a diagram to show that $(2x + 5)(x + 3)$ is equivalent to $2x^2 + 11x + 15$.

2. Match each quadratic expression that is written as a product with an equivalent expression that is expanded.

A. $(x + 2)(x + 6)$

B. $(2x + 8)(x + 2)$

C. $(x + 8)(x + 4)$

D. $(x + 2)(2x + 6)$

1. $x^2 + 12x + 32$
2. $2x^2 + 10x + 12$
3. $2x^2 + 12x + 16$
4. $x^2 + 8x + 12$

3. Select **all** expressions that are equivalent to $x^2 + 4x$.

A. $x(x + 4)$

B. $(x + 2)^2$

C. $(x + x)(x + 4)$

D. $(x + 2)^2 - 4$

E. $(x + 4)x$

4. Tyler drew a diagram to expand $(x + 5)(2x + 3)$.

a. Explain Tyler's mistake.

b. What is the correct expanded form of $(x + 5)(2x + 3)$?

	$2x$	3
x	$2x^2$	$3x$
5	$7x$	8

5. Use the diagram to show that:

$(x+4)(x+2)$ is equivalent to x^2+6x+8.

	x	2
x		
4		

$(x-10)(x-3)$ is equivalent to $x^2-13x+30$.

	x	-10
x		
-3		

6. What are the solutions to the equation $(x - a)(x + b) = 0$?

A. a and b *B.* $-a$ and $-b$ *C.* a and $-b$ *D.* $-a$ and b

7. Match each equation to an equivalent equation with a perfect square on one side.

A. $x^2+8x=2$
B. $x^2+10x=-13$
C. $x^2-14x=5$
D. $x^2+2x=0$
E. $x^2+4x-5=0$
F. $x^2-20x=-9$

1. $(x-7)^2=54$
2. $(x+5)^2=12$
3. $(x-10)^2=91$
4. $(x+4)^2=18$
5. $(x+1)^2=1$
6. $(x+2)^2=9$

8. Solve each equation by completing the square.

a. $x^2-6x+5=12$

b. $x^2-2x=8$

c. $11=x^2+4x-1$

d. $x^2-18x+60=-21$

Curated Practice Problem Set

Unit 6 Lesson 10 Cumulative Practice Problems

1. A quadratic function f is defined by $f(x) = (x - 7)(x + 3)$.

   a. Without graphing, identify the x-intercepts of the graph of f. Explain how you know.

   b. Expand $(x - 7)(x + 3)$ and use the expanded form to identify the y-intercept of the graph of f.

2. What are the x-intercepts of the graph of the function defined by $(x - 2)(2x + 1)$?

   A. $(2,0)$ and $(-1,0)$

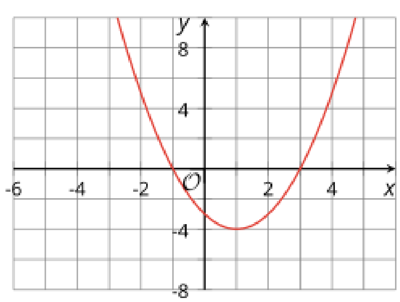
   B. $(2,0)$ and $\left(-\frac{1}{2}, 0\right)$

   C. $(-2,0)$ and $(1,0)$

   D. $(-2,0)$ and $(\frac{1}{2}, 0)$

3. Here is a graph that represents a quadratic function.

   Which expression could define this function?



   A. $(x + 3)(x + 1)$

   B. $(x + 3)(x - 1)$

   C. $(x - 3)(x + 1)$

   D. $(x - 3)(x - 1)$

4. a. What is the y-intercept of the graph of the equation $y = x^2 - 5x + 4$?

   b. An equivalent way to write this equation is $y = (x - 4)(x - 1)$. What are the x-intercepts of this equation's graph?

5. Noah said that if we graph $y = (x - 1)(x + 6)$, the x-intercepts will be at $(1,0)$ and $(-6,0)$. Explain how you can determine, without graphing, whether Noah is correct.

Lesson 13 Cumulative Practice Problems

1. Select **all** equations whose graphs have a vertex with x-coordinate 2.

   A. $y = (x-2)(x-4)$

   B. $y = (x-2)(x+2)$

   C. $y = (x-1)(x-3)$

   D. $y = x(x+4)$

   E. $y = x(x-4)$

2. Determine the x-intercepts and the x-coordinate of the vertex of the graph that represents each equation.

equation	**x-intercepts**	**x-coordinate of the vertex**
$y = x(x-2)$		
$y = (x-4)(x+5)$		
$y = -5x(3-x)$		

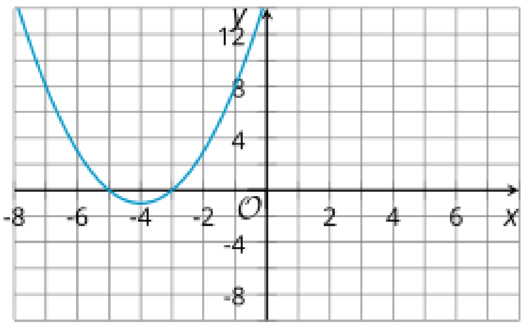
3. What are the x-intercepts of the graph of $y = (x-2)(x-4)$?

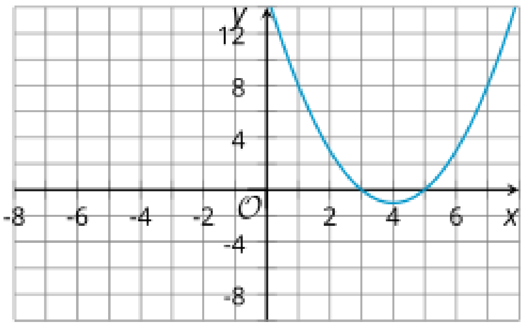
   b. Find the coordinates of another point on the graph. Show your reasoning.

   c. Sketch a graph of the equation $y = (x-2)(x-4)$.

4. Which one is the graph of the equation $y = (x - 3)(x + 5)$?

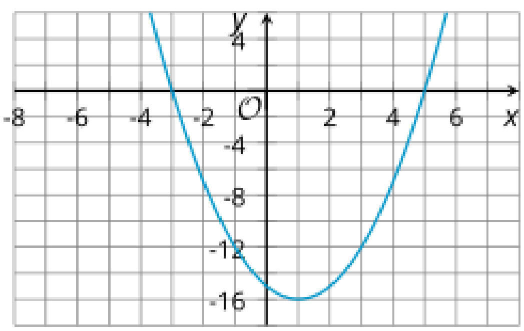
Graph A



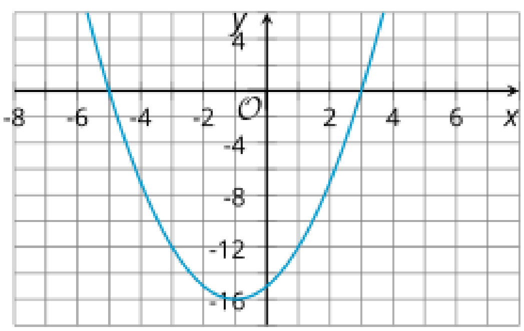


Graph B

Graph D



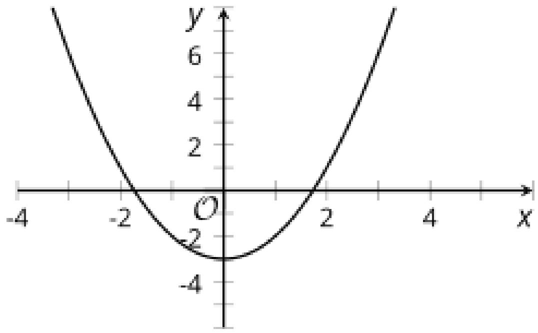
Graph C



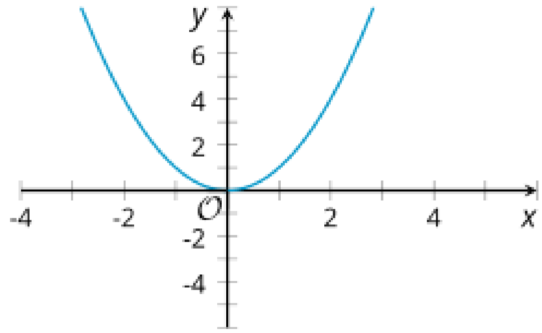
Lesson 14

1. Here are four graphs. Match each graph with a quadratic equation that it represents.

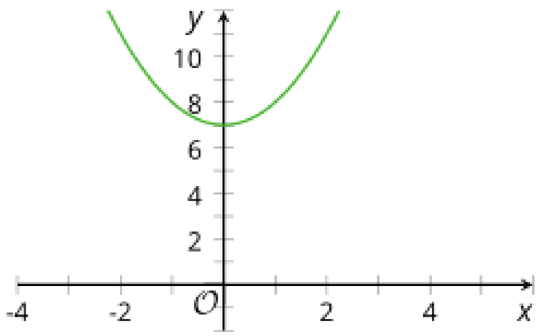
Graph A



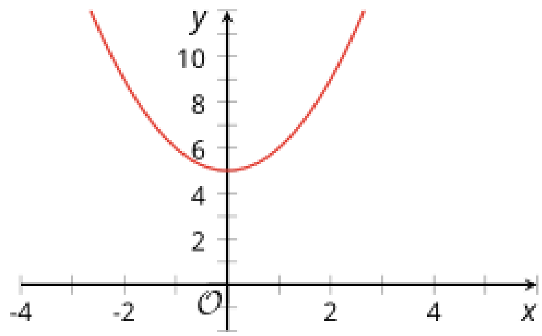
Graph B



Graph C



Graph D



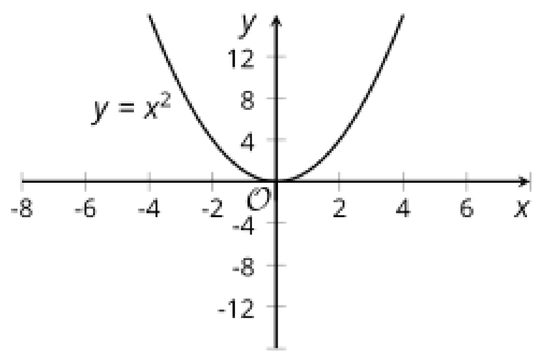
A. Graph A	1. $y = x^2$
B. Graph B	2. $y = x^2 + 5$
C. Graph C	3. $y = x^2 + 7$
D. Graph D	4. $y = x^2 - 3$

2. The two equations $y = (x+2)(x+3)$ and $y = x^2 + 5x + 6$ are equivalent.

   a. Which equation helps find the x-intercepts most efficiently?

   b. Which equation helps find the y-intercept most efficiently?

3. Here is a graph that represents $y = x^2$.



   On the same coordinate plane, sketch and label the graph that represents each equation:

   a. $y = x^2 - 4$

   b. $y = -x^2 + 5$

4. Select **all** equations whose graphs have a y-intercept with a positive y-coordinate.

   A. $y = x^2 + 3x - 2$

   B. $y = x^2 - 10x$

   C. $y = (x-1)^2$

   D. $y = 5x^2 - 3x - 5$

   E. $y = (x+1)(x+2)$

5. a. What are the x-intercepts of the graph that represents $y = (x+1)(x+5)$? Explain how you know.

   b. What is the x-coordinate of the vertex of the graph that represents $y = (x+1)(x+5)$? Explain how you know.

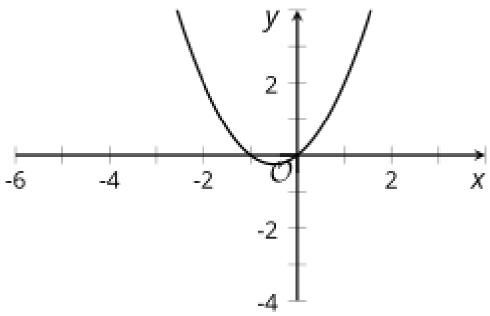
   c. Find the y-coordinate of the vertex. Show your reasoning.

   d. Sketch a graph of $y = (x+1)(x+5)$.

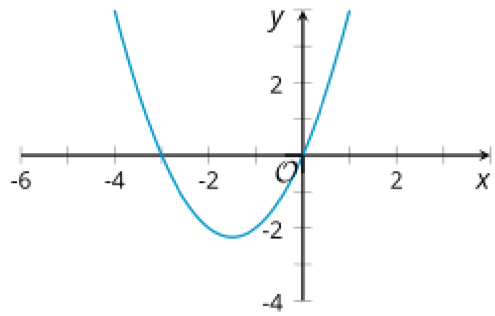
Lesson 15 Cumulative Practice Problems

1. Here are four graphs. Match each graph with the quadratic equation that it represents.

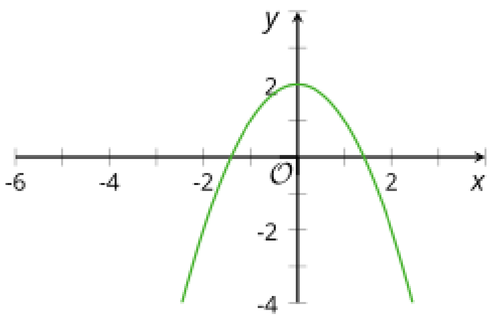
Graph A



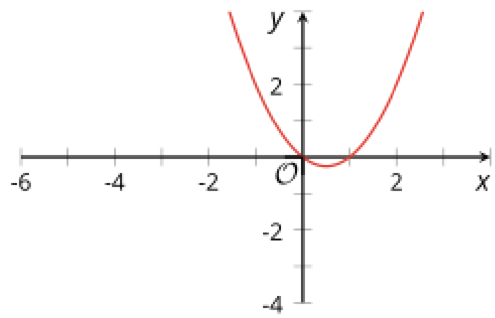
Graph B



Graph C



Graph D



A. Graph A
B. Graph B
C. Graph C
D. Graph D

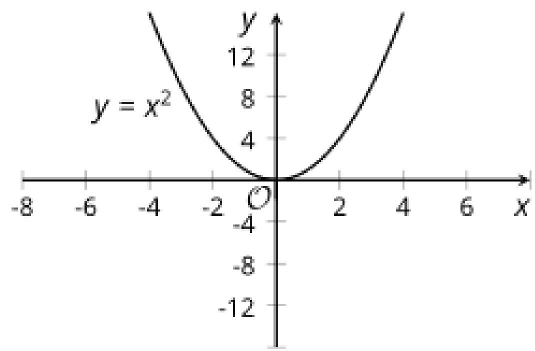
1. $y = x^2 + x$
2. $y = -x^2 + 2$
3. $y = x^2 - x$
4. $y = x^2 + 3x$

2. Complete the table without graphing the equations.

equation	x-intercepts	x-coordinate of the vertex
$y = x^2 + 12x$		
$y = x^2 - 3x$		
$y = -x^2 + 16x$		
$y = -x^2 - 24x$		

3. Here is a graph that represents $y = x^2$.

   a. Describe what would happen to the graph if the original equation were changed to $y = x^2 - 6x$. Predict the x- and y-intercepts of the graph and the quadrant where the vertex is located.



   b. Sketch the graph of the equation $y = x^2 - 6x$ on the same coordinate plane as $y = x^2$.

4. Select **all** equations whose graph opens upward.

   A. $y = -x^2 + 9x$

   B. $y = 10x - 5x^2$

   C. $y = (2x - 1)^2$

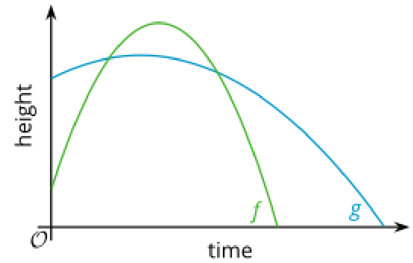
   D. $y = (1 - x)(2 + x)$

   E. $y = x^2 - 8x - 7$

Lesson 16 Cumulative Practice Problems

1. Here are graphs of functions f and g.

   Each represents the height of an object being launched into the air as a function of time.



   a. Which object was launched from a higher point?

   b. Which object reached a higher point?

   c. Which object was launched with the higher upward velocity?

   d. Which object landed last?

2. *Technology required*. The function h given by $h(t) = (1 - t)(8 + 16t)$ models the height of a ball in feet, t seconds after it was thrown.

   a. Find the zeros of the function. Show or explain your reasoning.

   b. What do the zeros tell us in this situation? Are both zeros meaningful?

   c. From what height is the ball thrown? Explain your reasoning.

   d. About when does the ball reach its highest point, and about how high does the ball go? Show or explain your reasoning.

3. The height in feet of a thrown football is modeled by the equation $f(t) = 6 + 30t - 16t^2$, where time t is measured in seconds.

   a. What does the constant 6 mean in this situation?

   b. What does the $30t$ mean in this situation?

   c. How do you think the squared term $-16t^2$ affects the value of the function f? What does this term reveal about the situation?

4. The height in feet of an arrow is modeled by the equation $h(t) = (1 + 2t)(18 - 8t)$, where t is seconds after the arrow is shot.

   a. When does the arrow hit the ground? Explain or show your reasoning.

   b. From what height is the arrow shot? Explain or show your reasoning.

5. Two objects are launched into the air.

   - The height, in feet, of Object A is given by the equation $f(t) = 4 + 32t - 16t^2$.

   - The height, in feet, of the Object B is given by the equation $g(t) = 2.5 + 40t - 16t^2$. In both functions, t is seconds after launch.

   a. Which object was launched from a greater height? Explain how you know.

   b. Which object was launched with a greater upward velocity? Explain how you know.

Lesson 17 Cumulative Practice Problems

1. Select **all** of the quadratic expressions in vertex form.

   A. $(x-2)^2+1$

   B. x^2-4

   C. $x(x+1)$

   D. $(x+3)^2$

   E. $(x-4)^2+6$

2. Here are two equations. One defines function m and the other defines function p.

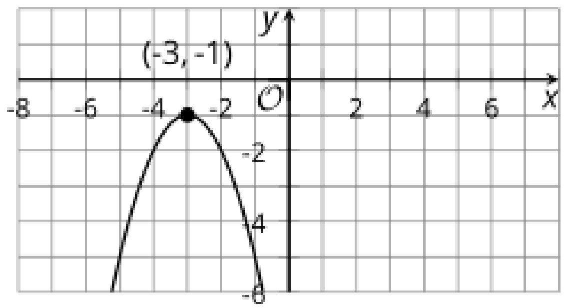
   $m(x) = x(x+6)$ $\qquad$ $p(x) = (x+3)^2 - 9$

   a. Show that the expressions defining m and p are equivalent.

   b. What is the vertex of the graph of m? Explain how you know.

   c. What are the x-intercepts of the graph of p? Explain how you know.

3. Which equation is represented by the graph?



A. $y = (x - 1)^2 + 3$

B. $y = (x - 3)^2 + 1$

C. $y = -(x + 3)^2 - 1$

D. $y = -(x - 3)^2 + 1$

4. For each equation, write the coordinates of the vertex of the graph that represents the equation.

a. $y = (x - 3)^2 + 5$

b. $y = (x + 7)^2 + 3$

c. $y = (x - 4)^2$

d. $y = x^2 - 1$

e. $y = 2(x + 1)^2 - 5$

f. $y = -2(x + 1)^2 - 5$

Lesson 18 Cumulative Practice Problems

1. Which equation can be represented by a graph with a vertex at (1,3)?

   A. $y = (x - 1)^2 + 3$

   B. $y = (x + 1)^2 + 3$

   C. $y = (x - 3)^2 + 1$

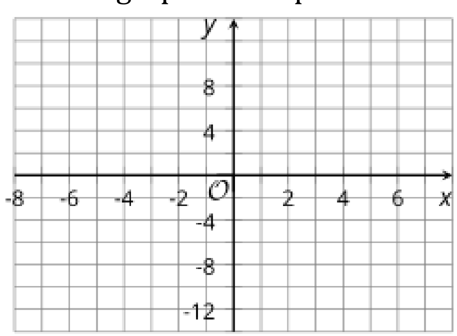
   D. $y = (x + 3)^2 + 1$

2. a. Where is the vertex of the graph that represents $y = (x - 2)^2 - 8$?

   b. Where is the y-intercept? Explain how you know.

   c. Identify one other point on the graph of the equation. Explain or show how you know.

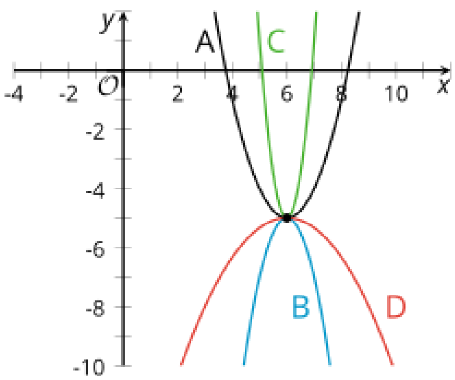
   d. Sketch a graph that represents the equation.



3. The function v is defined by $v(x) = \frac{1}{2}(x+5)^2 - 7$.

Without graphing, determine if the vertex of the graph representing v shows the minimum or maximum value of the function. Explain how you know.

4. Match each graph to an equation that represents it.

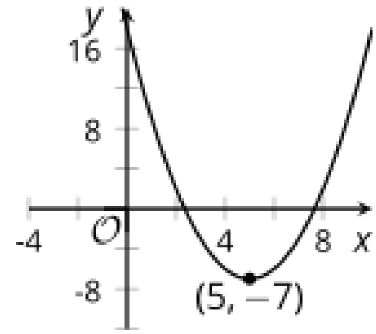


A. Graph A
B. Graph B
C. Graph C
D. Graph D

1. $y = -2(x-6)^2 - 5$
2. $y = (x-6)^2 - 5$
3. $y = 6(x-6)^2 - 5$
4. $y = -\frac{1}{3}(x-6)^2 - 5$

Lesson 19 Cumulative Practice Problems

1. Here the graph of quadratic function f.



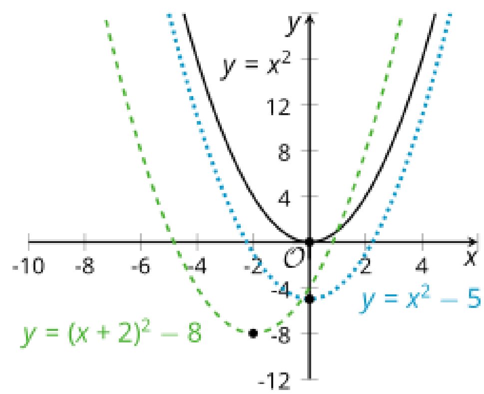
Andre uses the expression $(x-5)^2+7$ to define f.

Noah uses the expression $(x+5)^2-7$ to define f.

Do you agree with either of them? Explain your reasoning.

2. Here are the graphs of $y=x^2$, $y=x^2-5$, and $y=(x+2)^2-8$.

   a. How do the 3 graphs compare?



   b. How does the -5 in x^2-5 affect the graph?

   c. How does the +2 and the -8 in $(x+2)^2-8$ affect the graph?

3. Which equation represents the graph of $y=x^2+2x-3$ moved 3 units to the left?

   A. $y=x^2+2x-6$

   B. $y=(x+3)^2+2x-3$

   C. $y=(x+3)^2+2(x+3)$

   D. $y=(x+3)^2+2(x+3)-3$

4. Select **all** the equations with a graph whose vertex has *both* a positive x- and a positive y-coordinate.

   A. $y = x^2$

   B. $y = (x - 1)^2$

   C. $y = (x - 3)^2 + 2$

   D. $y = 2(x - 4)^2 - 5$

   E. $y = 0.5(x + 2)^2 + 6$

   F. $y = -(x - 4)^2 + 3$

   G. $y = -2(x - 3)^2 + 1$

5. The height in feet of a soccer ball is modeled by the equation $g(t) = 2 + 50t - 16t^2$, where time t is measured in seconds after it was kicked.

   a. How far above the ground was the ball when kicked?

   b. What was the initial upward velocity of the ball?

   c. Why is the coefficient of the squared term negative?

Wrap Up

1. The function f is given by $f(x) = x^2 - 2x$. Which statement is true about the graph of f?

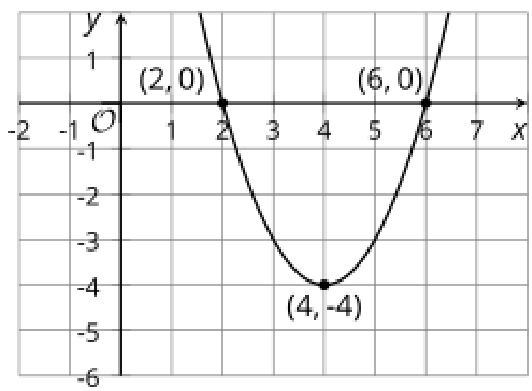
   A. The graph has a y-intercept at $(2,0)$.

   B. The coordinates of the vertex are $(1,1)$.

   C. The graph of the function opens downward.

   D. The x-intercepts are at $(0,0)$ and $(2,0)$.

2. Here is the graph of a quadratic function f.



Select **all** equations that could define the function f.

A. $f(x) = -x^2 + 8x - 12$

B. $f(x) = x^2 - 8x + 12$

C. $f(x) = (x + 2)(x + 6)$

D. $f(x) = (x - 4)^2 + 4$

E. $f(x) = (x - 4)^2 - 4$

F. $f(x) = (x - 2)(x - 6)$

3. A quadratic function f is given by $f(x) = ax^2 + bx + c$ where a is not 0. Select **all** the statements that *must* be true about the graph of f.

   A. The y-intercept of the graph is at $(0, c)$.

   B. The graph has an x-intercept at $(c, 0)$.

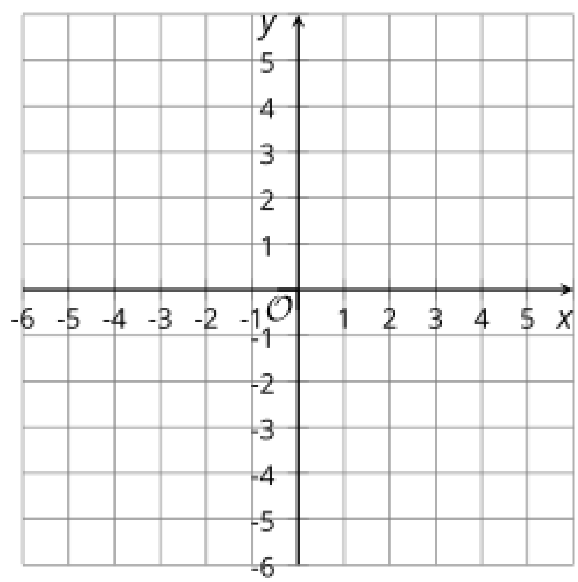
   C. When $a < 0$ the graph opens downward.

   D. The graph has two x-intercepts.

   E. If $b = 0$, then the vertex is on the y-axis.

4. A quadratic function is defined by $p(x) = (x - 1)(x + 3)$.

   Graph the function on the coordinate plane and label the coordinates of the x-intercepts, the y-intercept, and the vertex.



5. A quadratic function is defined by $g(x) = (x + 4)^2 + 7$.

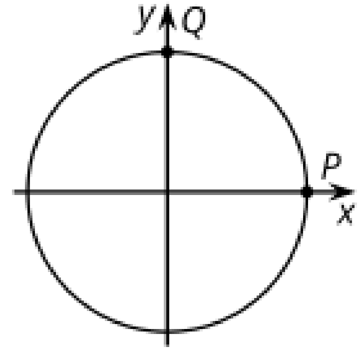
   a. What is the vertex of the graph of function g?

   b. Does the vertex represent the minimum value or the maximum value of the function? Explain or show how you know.

   c. If you were to shift this graph 6 units down from where it is now, what would be the equation represented by the new graph?

Lesson 11 Review Problems – use previous videos if you need help.

1. A rotation takes P to Q. What could be the measure of the angle of rotation in radians? Select **all** that apply.



A. $\frac{3\pi}{2}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

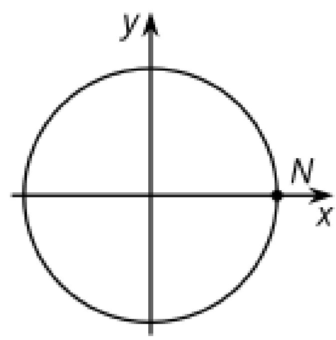
D. $\frac{5\pi}{2}$

E. $\frac{5\pi}{4}$

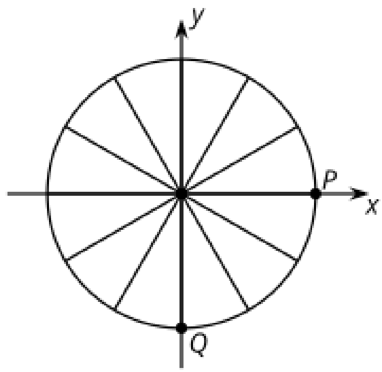
2. a. A $\frac{2\pi}{3}$ radian rotation takes N to P. Label P.

   b. A $\frac{7\pi}{6}$ radian rotation takes N to Q. Label Q.

   c. A $\frac{25\pi}{6}$ radian rotation takes N to R. Label R.



3. Here is a wheel with radius 1 foot.



a. List three different counterclockwise angles the wheel can rotate so that point P ends up at position Q.

b. How many feet does the wheel roll for each of these angles?

4. The point P on the unit circle is in the 0 radian position.

a. Which counterclockwise rotations take P back to itself? Explain how you know.

b. Which counterclockwise rotations take P to the opposite point on the unit circle? Explain how you know.

Video on converting to an angles between 0 and 2π to find trig values.

Lesson 12

1. For which of these angles is the sine negative? Select **all** that apply.

   A. $-\frac{\pi}{4}$

   B. $-\frac{\pi}{3}$

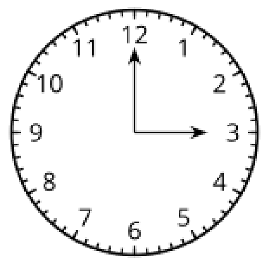
   C. $-\frac{2\pi}{3}$

   D. $-\frac{4\pi}{3}$

   E. $-\frac{11\pi}{6}$

2. The clock reads 3:00 p.m.

   Which of the following are true? Select **all** that apply.



   A. In the next hour, the minute hand moves through an angle of 2π radians.

   B. In the next 5 minutes, the minute hand will move through an angle of $-\frac{\pi}{6}$ radians.

   C. After the minute hand moves through an angle of $-\pi$ radians, it is 3:30 p.m.

   D. When the hour hand moves through an angle of $-\frac{\pi}{6}$ radians, it is 4:00 p.m.

   E. The angle the minute hand moves through is 12 times the angle the hour hand moves through.

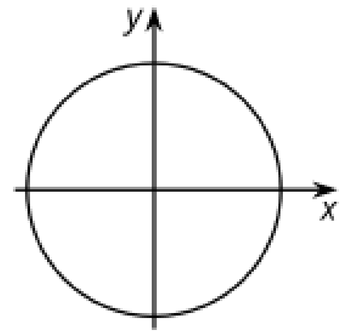
3. Plot each point on the unit circle.

a. $A = (\cos(-\frac{\pi}{4}), \sin(-\frac{\pi}{4}))$

b. $B = (\cos(2\pi), \sin(2\pi))$

c. $C = \left(\cos(\frac{16\pi}{3}), \sin(\frac{16\pi}{3})\right)$

d. $D = \left(\cos(-\frac{16\pi}{3}), \sin(-\frac{16\pi}{3})\right)$



4. Which of these statements are true about the function f given by $f(\theta) = \sin(\theta)$? Select **all** that apply.

A. The graph of f meets the θ-axis at $0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

B. The value of f always stays the same when π radians is added to the input.

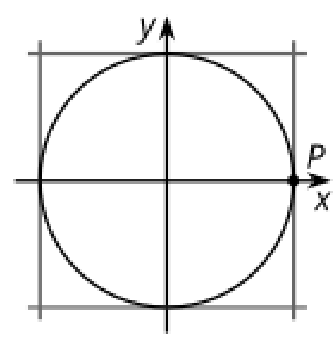
C. The value of f always stays the same when 2π radians is added to the input.

D. The value of f always stays the same when -2π radians is added to the input.

E. The graph of f has a maximum when $\theta = \frac{5\pi}{2}$ radians.

5. Here is a unit circle with a point P at (1,0).

For each positive angle of rotation of the unit circle around its center listed, indicate on the unit circle where P is taken, and give a negative angle of rotation which takes P to the same location.



a. $A, \frac{\pi}{4}$ radians

b. $B, \frac{\pi}{2}$ radians

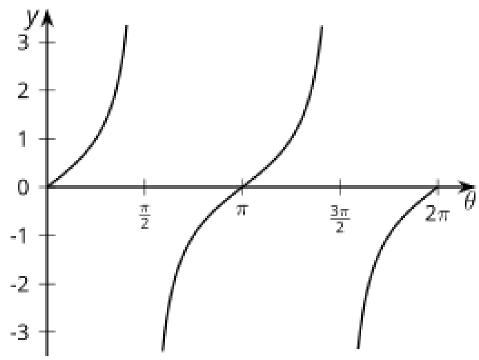
c. C, π radians

d. $D, \frac{3\pi}{2}$ radians

Video on graphing tangent on a θ, f(θ) coordinate plane.

Lesson 13 Cumulative Practice Problems

1. Here is a graph of f given by $f(\theta) = \tan(\theta)$.



   a. Are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ in the domain of f? Explain how you know.

   b. What are the θ-intercepts of the graph of f? Explain how you know.

2. The function f is given by $f(\theta) = \tan(\theta)$. Which of the statements are true? Select **all** that apply.

   A. f is a periodic function

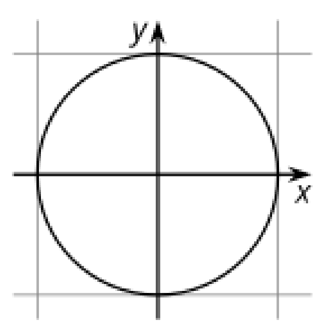
   B. The domain of f is all real numbers.

   C. The range of f is all real numbers.

   D. The period of f is 2π.

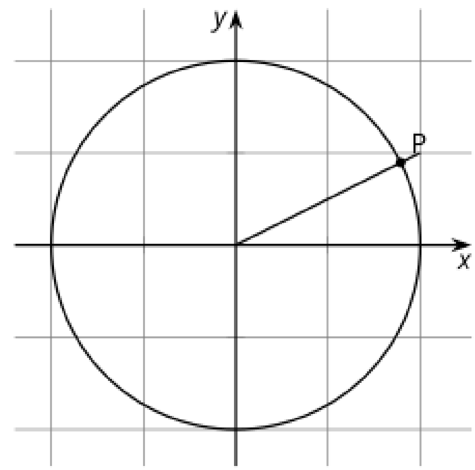
   E. The period of f is π.

3. Here is the unit circle.



   If $\tan(a) > 1$ where could angle a be on the unit circle?

4. Here is a point on the unit circle.



a. Explain why the line going through (0,0) and P has slope $\frac{1}{2}$.

b. What is the tangent of the angle represented by P? Explain how you know.

5. For which angles θ between 0 and 2π is $\cos(\theta) < 0$? Explain how you know.

Midline, Amplitude, and Extreme Values

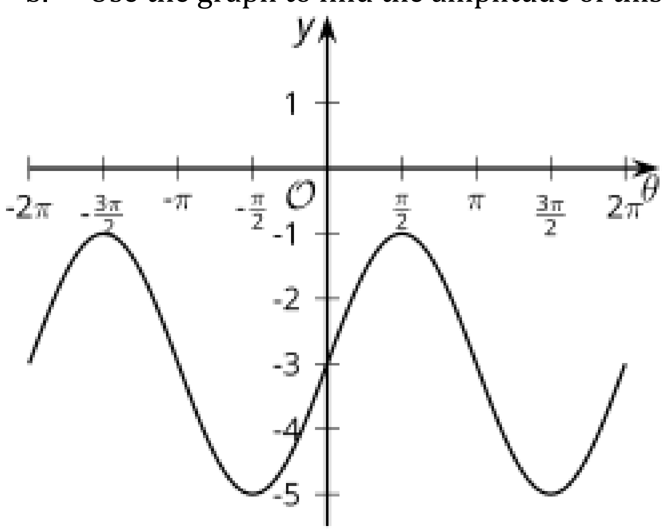
Lesson 14

1. For each trigonometric function, indicate the amplitude and midline.

   a. $y = 2\sin(\theta)$
   b. $y = \cos(\theta) - 5$
   c. $y = 1.4\sin(\theta) + 3.5$

2. Here is a graph of the equation $y = 2\sin(\theta) - 3$.

   a. Indicate the midline on the graph.
   b. Use the graph to find the amplitude of this sine equation.

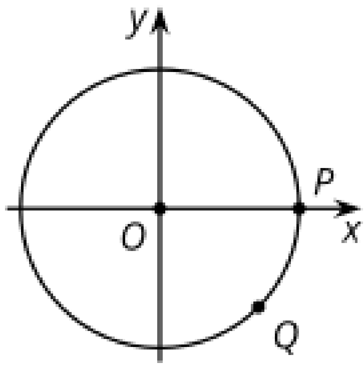


3. Select **all** trigonometric functions with an amplitude of 3.

   a. $y = 3\sin(\theta) - 1$
   b. $y = \sin(\theta) + 3$
   c. $y = 3\cos(\theta) + 2$
   d. $y = \cos(\theta) - 3$
   e. $y = 3\sin(\theta)$
   f. $y = \cos(\theta - 3)$

4. The measure of angle θ, in radians, satisfies $\sin(\theta) < 0$. If θ is between 0 and 2π what can you say about the measure of θ?

5. Which rotations, with center O, take P to Q? Select **all** that apply.



a. $\frac{3\pi}{4}$ radians

b. $\frac{15\pi}{4}$ radians

c. $\frac{7\pi}{4}$ radians

d. $\frac{11\pi}{4}$ radians

e. $\frac{23\pi}{4}$ radians

Horizontal translations

Vertical translation

Remember that the same rules that apply to sine also apply to cosine and tangent.

Lesson 15

1. These equations model the vertical position, in feet above the ground, of a point at the end of a windmill blade. For each function, indicate the height of the windmill and the length of the windmill blades.

   a. $y = 5\sin(\theta) + 10$

   b. $y = 8\sin(\theta) + 20$

   c. $y = 4\sin(\theta) + 15$

2. Which expression takes the same value as $\cos(\theta)$ when $\theta = 0, \frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$?

   A. $\sin\left(\theta - \frac{\pi}{2}\right)$

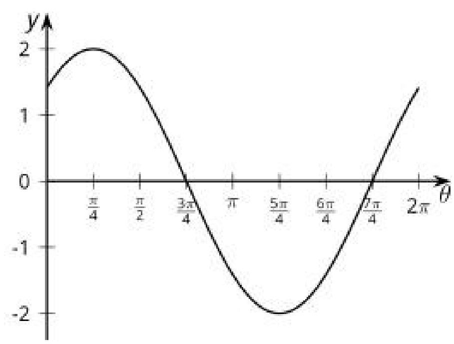
   B. $\sin\left(\theta + \frac{\pi}{2}\right)$

   C. $\sin(\theta + \pi)$

   D. $\sin(\theta - \pi)$

3. Here is a graph of a trigonometric function.

   Which equation does the graph represent?



   A. $y = 2\sin(\theta)$

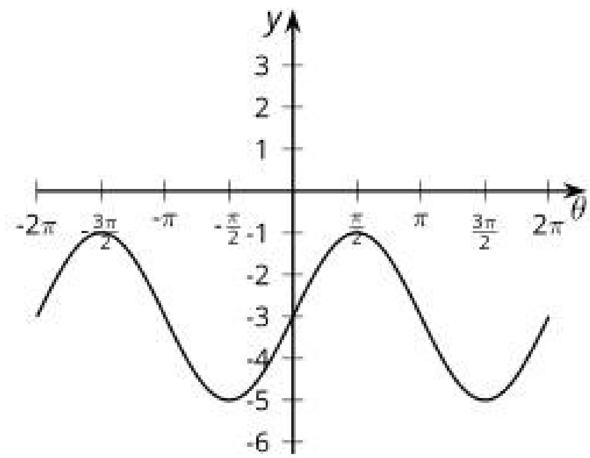
   B. $y = 2\cos\left(\theta + \frac{\pi}{4}\right)$

   C. $y = 2\sin\left(\theta - \frac{\pi}{4}\right)$

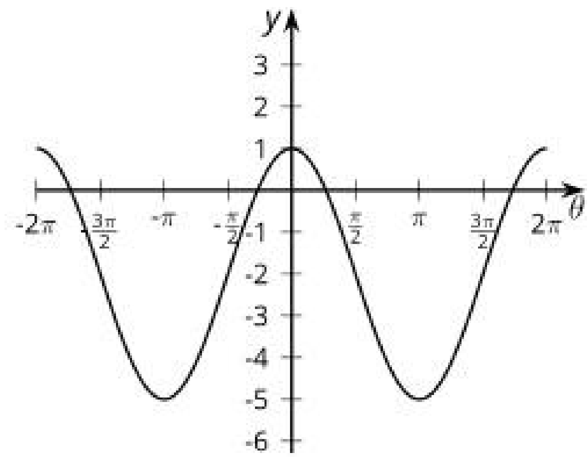
   D. $y = 2\cos\left(\theta - \frac{\pi}{4}\right)$

4. Match the trigonometric expressions with their graphs.

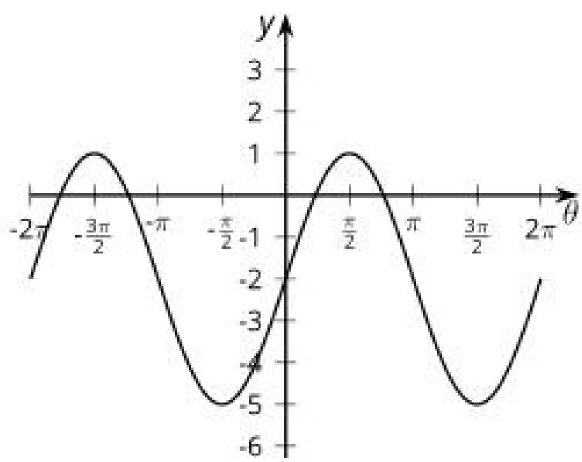
Graph 1



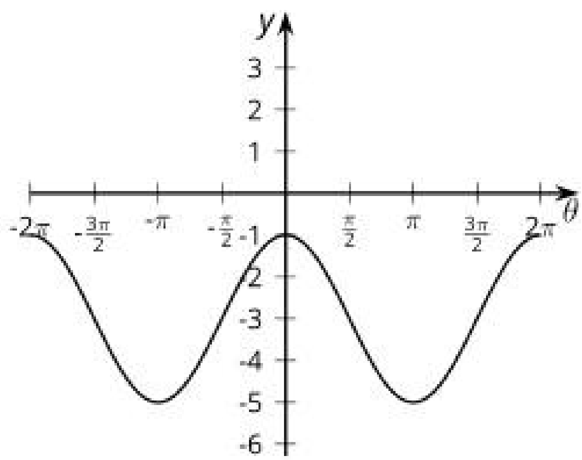
Graph 2



Graph 3



Graph 4



A. $3\cos(\theta) - 2$

B. $2\cos(\theta) - 3$

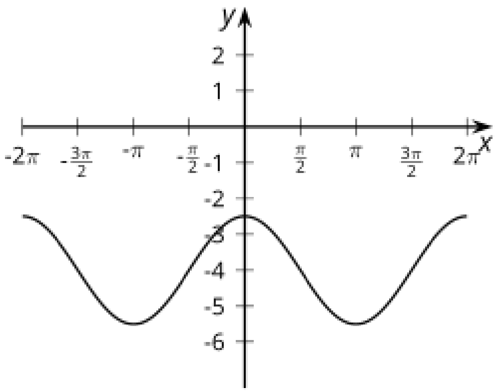
C. $3\sin(\theta) - 2$

D. $2\sin(\theta) - 3$

1. Graph 1
2. Graph 2
3. Graph 3
4. Graph 4

Lesson 16 Cumulative Practice Problems

1. Here is a graph of a trigonometric function. Which equation could define this function?



A. $y = 1.5\sin(x) - 4$

B. $y = 1.5\cos(x) - 4$

C. $y = -4\sin(1.5x)$

D. $y = -4\cos(1.5x)$

2. Select **all** the functions that have period π.

A. $y = \cos\left(\frac{x}{2}\right)$

B. $y = \sin\left(\frac{x}{2}\right)$

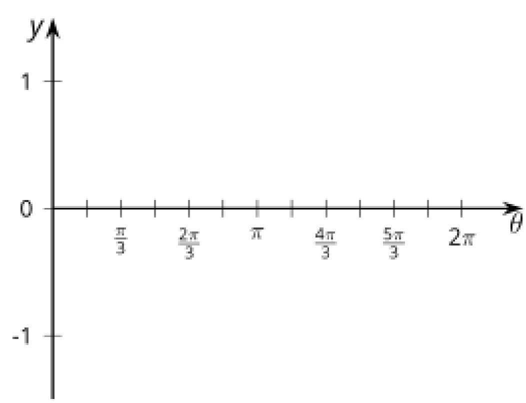
C. $y = \cos(x)$

D. $y = \cos(2x)$

E. $y = \sin(2x)$

3. a. Sketch a graph of $a(\theta) = \cos(3\theta)$.

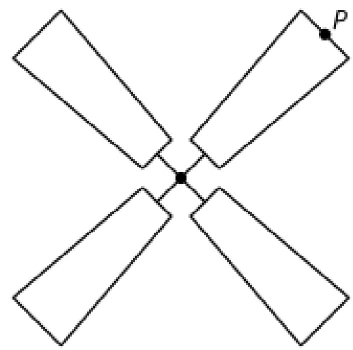
b. Compare the graph of a to the graph of $b(\theta) = \cos(\theta)$. How are the two graphs alike? How are they different?



4.The functions f and g are given by $f(x) = \cos(x)$ and $g(x) = \cos(5x)$. How are the graphs of f and g related?

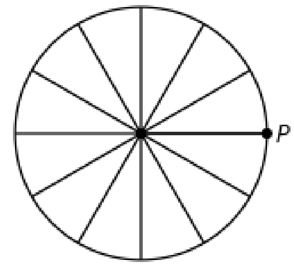
5.Here is a point at the tip of a windmill blade. The center of the windmill is 6 feet off the ground and the blades are 1.5 feet long.

Write an equation giving the height h of the point P after the windmill blade rotates by an angle of a. Point P is currently rotated $\frac{\pi}{4}$ radians from the point directly to the right of the center of the windmill.



Lesson 16 Cumulative Practice Problems

1. A wheel rotates three times per second in a counterclockwise direction. The center of the wheel does not move.

   What angle does the point P rotate through in one second?

   A. $\frac{2\pi}{3}$ radians

   B. 2π radians

   C. 3π radians

   D. 6π radians

2. A bicycle wheel is spinning in place. The vertical position of a point on the wheel, in inches, is described by the function $f(t) = 13.5\sin(5 \bullet 2\pi t) + 20$. Time $\boldsymbol{t}$ is measured in seconds.

   a. What is the meaning of 13.5 in this context?

   b. What is the meaning of 5 in this context?

   c. What is the meaning of 20 in this context?

3. Each expression describes the vertical position, in feet off the ground, of a carriage on a Ferris wheel after t minutes. Which function describes the largest Ferris wheel?

A. $100\sin\left(\frac{2\pi t}{20}\right) + 110$

B. $100\sin\left(\frac{2\pi t}{30}\right) + 110$

C. $200\sin\left(\frac{2\pi t}{30}\right) + 210$

D. $250\sin\left(\frac{2\pi t}{20}\right) + 260$

4. Which trigonometric function has period 5?

A. $f(x) = \sin\left(\frac{1}{5}x\right)$

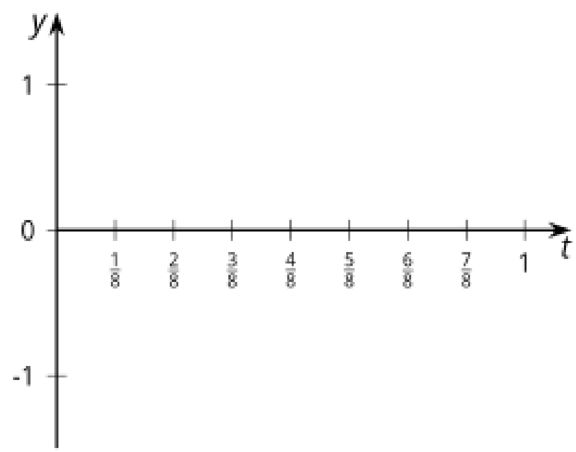
B. $f(x) = \sin(5x)$

C. $f(x) = \sin\left(\frac{5}{2\pi}x\right)$

D. $f(x) = \sin\left(\frac{2\pi}{5}x\right)$

5. a. What is the period of the function f given by $f(t) = \cos(4\pi t)$? Explain how you know.

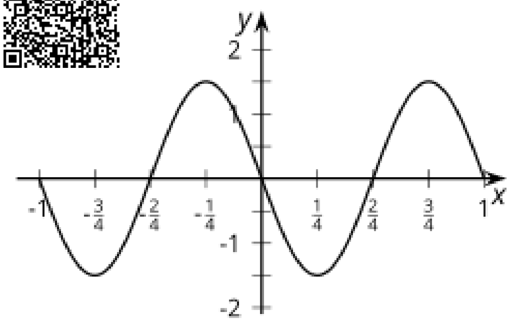
b. Sketch a graph of f.



Lesson 18 Cumulative Practice Problems

1. Here is the graph of a trigonometric function.

   Which equation has this graph? Select **all** that apply.



   A. $y = \frac{3}{2}\cos\left(2\pi x - \frac{\pi}{2}\right)$

   B. $y = -\frac{3}{2}\sin(2\pi x)$

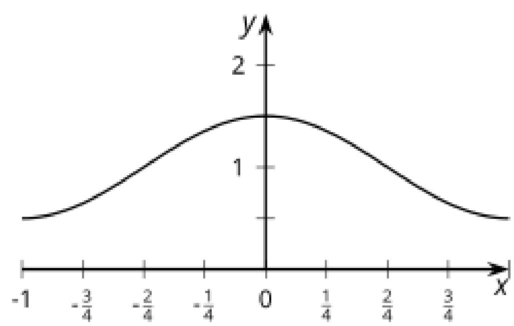
   C. $y = \frac{3}{2}\cos(2\pi x)$

   D. $y = \frac{3}{2}\cos\left(2\pi x + \frac{\pi}{2}\right)$

   E. $y = \frac{3}{2}\sin(2\pi x + \pi)$

2. Here is the graph of a trigonometric function.

   Which equation has this graph?



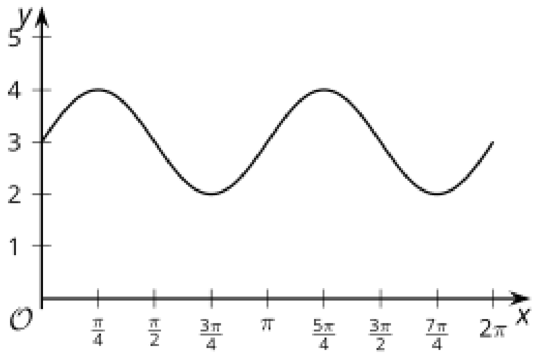
   A. $y = \cos(x) + 1$

   B. $y = \frac{1}{2}\cos(x) + 1$

   C. $y = \frac{1}{2}\cos(\pi \bullet x) + 1$

   D. $y = \frac{1}{2}\cos(2\pi \bullet x) + 1$

3. Here is the graph of a trigonometric function.



a. Find a trigonometric function f that has this graph. Explain your reasoning.

b. The graph is translated right by $\frac{\pi}{2}$ so it has a minimum value at $x = 0$, then stretched horizontally so its period is 3 times greater than the period of f. Find a trigonometric function g that has this new graph. Explain your reasoning.

4. The function f is given by $f(x) = 4 + 2\sin(\pi x)$. The graph of g is the graph of f

5. translated left by $\frac{\pi}{2}$ and translated vertically by -1. Which expression defines g?

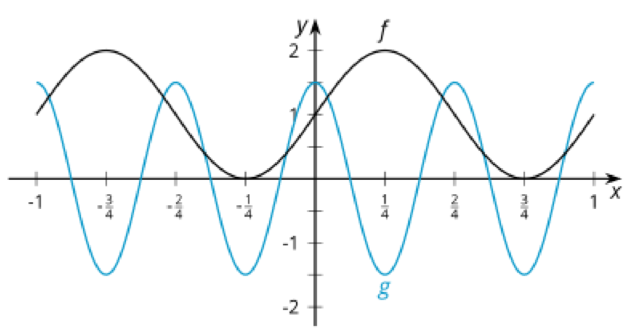
A. $5 + 2\sin\left(\pi x + \frac{\pi}{2}\right)$

B. $3 + 2\sin\left(\pi x + \frac{\pi}{2}\right)$

C. $3 + 2\sin\left(\pi\left(x - \frac{\pi}{2}\right)\right)$

D. $3 + 2\sin\left(\pi\left(x + \frac{\pi}{2}\right)\right)$

6. Here are graphs of trigonometric functions f and g. What transformations can be applied to the graph of f to get the graph of g? Make sure to list them in the order they are applied.

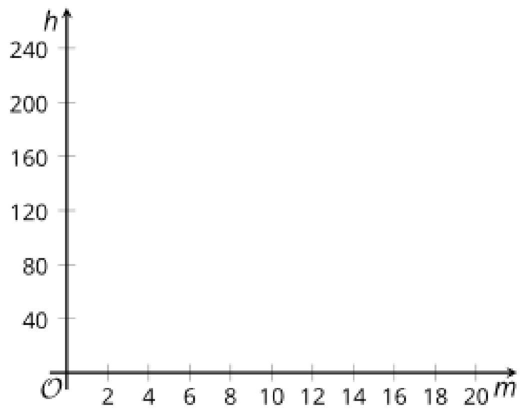


Lesson 19 Cumulative Practice Problems

1. Jada is riding on a Ferris wheel. Her height, in feet, is modeled by the function

   $h(m) = 100\sin\left(-\frac{\pi}{2} + \frac{2\pi m}{10}\right) + 110$, where m is the number of minutes since she got on the ride.

   a. How many minutes does it take the Ferris wheel to make one full revolution? Explain how you know.

   b. What is the radius of the Ferris wheel? Explain how you know.

   c. Sketch a graph of h.

2. The vertical position, in feet, of the point P on a windmill is represented by $y = 5\sin\left(\frac{2\pi t}{3}\right) + 20$, where t is the number of seconds after the windmill started turning at a constant speed. Select **all** the true statements.

   A. The windmill blades are 5 feet long.

   B. The windmill blades make 5 revolutions per second.

   C. The midline for the graph of the equation is 20.

   D. The windmill makes one revolution every 3 seconds.

   E. The windmill makes 3 revolutions per second.

3. A seat on a Ferris wheel travels 250π feet in one full revolution. How many feet is the carriage from the center of the Ferris wheel?

   A. $\frac{125}{\pi}$

   B. $\frac{250}{\pi}$

   C. 125

   D. 250

4. A carousel has a radius of 20 feet. The carousel makes 8 complete revolutions.

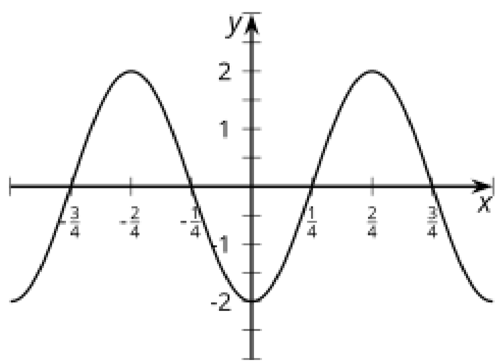
   a. How many feet does a person on the carousel travel during these 8 revolutions?

   b. What angle does the carousel travel through?

   c. What is the relationship between the angle of rotation and the distance traveled on this carousel? Explain your reasoning.

5. Here is the graph of a trigonometric function.

   Which equation has this graph?



   A. $y = -2\sin(2x)$

   B. $y = 2\sin\left(2\pi\left(x + \frac{1}{4}\right)\right)$

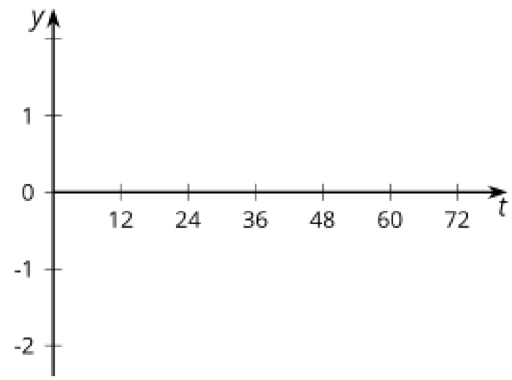
   C. $y = 2\sin\left(2\pi\left(x - \frac{1}{4}\right)\right)$

   D. $y = 2\sin\left(2\pi\left(x - \frac{\pi}{4}\right)\right)$

Lesson 20 Practice Problems

1. The water level f, in feet, at a certain beach is modeled by the function $f(t) = 2sin\left(\frac{2\pi t}{24}\right)$, where t is the number of hours since the level was measured.

   a. What is the amplitude of the function? What does it mean in this context?



   b. What is the period of the function? What does it mean in this context?

   c. Sketch a graph of the function over 72 hours.

2. The amount of the Moon visible d days after November 30 is modeled by the equation $f(d) = 0.5cos\left(\frac{2\pi\bullet(d-6)}{30}\right) + 0.5$. Select **all** the statements that are true for this model.

   A. The model predicts a full moon on December 6.

   B. The model predicts that there will be two full moons in December.

   C. The model predicts that none of the Moon will be visible on December 21.

   D. The model predicts that more than half of the Moon will be visible on December 13.

   E. The model predicts that there is a full moon every 30 days.

Lesson 11 Cumulative Practice Problems – Review

Use videos from the last two weeks to assist you if you get stuck

1. Select **all** equations that are parallel to the line $2x + 5y = 8$.

   A. $y = \frac{2}{5}x + 4$

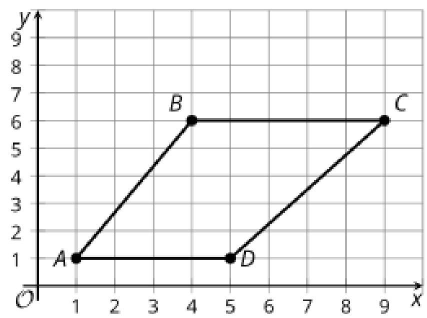
   B. $y = -\frac{2}{5}x + 4$

   C. $y - 2 = \frac{5}{2}(x + 1)$

   D. $y - 2 = -\frac{2}{5}(x + 1)$

   E. $10x + 5y = 40$

2. Prove that $ABCD$ is not a parallelogram.



3. Write an equation of a line that passes through $(-1,2)$ and is parallel to a line with x-intercept $(3,0)$ and y-intercept $(0,1)$.

4. Write an equation of the line with slope $\frac{2}{3}$ that goes through the point $(-2,5)$.

5. Write an equation for a line that passes through the origin and is perpendicular to $y = 5x - 2$.

6. Match each line with a perpendicular line.

   A. $y = 5x + 2$

   B. $y - 2.25 = -2(x - 2)$

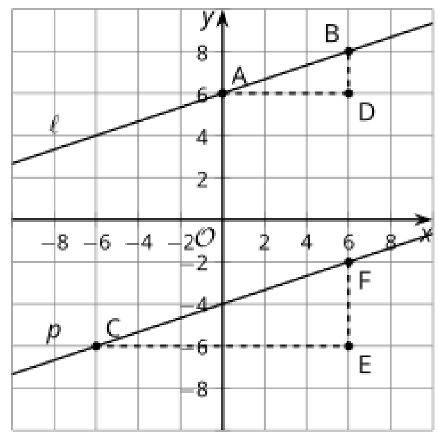
   C. the line through $(-1,5)$ and $(1,9)$

   1. the line through $(2,12)$ and $(17,9)$

   2. $y = -\frac{1}{2}x + 5$

   3. $2x - 4y = 10$

F.



7. Lines ℓ and p are parallel. Select **all** true statements.

    A. Triangle ADB is similar to triangle CEF.

    B. Triangle ADB is congruent to triangle CEF.

    C. The slope of line ℓ is equal to the slope of line p.

    D. Δ ABD is similar to ΔCFE

    E. line ℓ is perpendicular to line CE

Proving on the Coordinate Plane

Lesson 12 Cumulative Practice Problems

1. A quadrilateral has vertices $A = (0,0)$, $B = (1,3)$, $C = (0,4)$, and $D = (-1,1)$. Prove that $ABCD$ is a parallelogram.

2. A rhombus has vertices at $(0,0)$, $(5,0)$, $(3,4)$, and $(8,4)$.

   a. Find the slopes of the 2 diagonals of the rhombus.

   b. What do the slopes tell you about the diagonals in this rhombus?

3. a. Show that the triangle formed with vertices at $(0,0)$, $(4,3)$, and $(-2,11)$ is a right triangle.

   b. Find the area of the triangle.

Lesson 13 Cumulative Practice Problems

Coordinate Proof

1. Consider the parallelogram with vertices at (0,0), (4,0), (2,3), and (6,3). Where do the diagonals of this parallelogram intersect?

   A. (3,1.5) B. (4,2) C. (2,4) D. (3.5,3)

2. What is the midpoint of the line segment with endpoints $(1,-2)$ and $(9,8)$?

   A. (3,5) B. (4,3) C. (5,3) D. (5,5)

3. A quadrilateral has vertices $A=(0,0)$, $B=(2,4)$, $C=(0,5)$, and $D=(-2,1)$. Prove that $ABCD$ is a rectangle.

4. A quadrilateral has vertices $A=(0,0)$, $B=(1,3)$, $C=(0,4)$, and $D=(-1,1)$. Select the most precise classification for quadrilateral $ABCD$.

   A. quadrilateral

   B. parallelogram

   C. rectangle

   D. square

5. A trapezoid is a quadrilateral with at least one pair of parallel sides. Show that the quadrilateral formed by the vertices (0,0), (5,2), (10,10), and (0,6) is a trapezoid.

Putting it All Together – Use the links to videos from previous lessons if you need help.

1. Line m is represented by the equation $y + 2 = \frac{3}{2}(x + 4)$. Select **all** equations that represent lines perpendicular to line m.

A. $y = -\frac{3}{2}x + 4$

B. $y = -\frac{2}{3}x + 4$

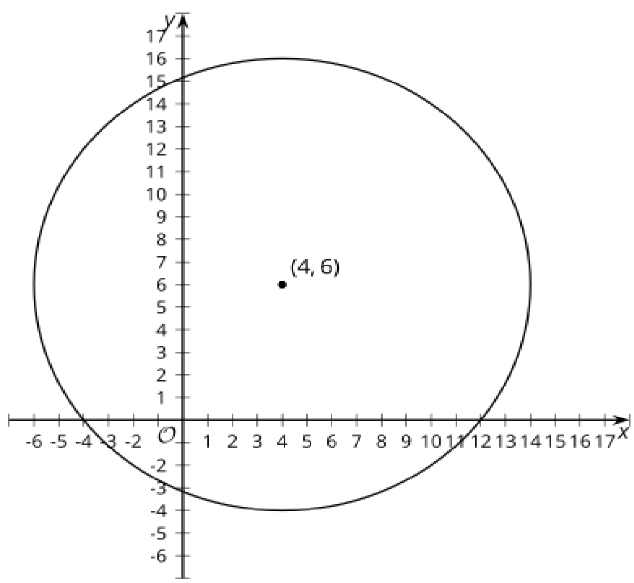
C. $y = \frac{2}{3}x + 4$

D. $y = \frac{3}{2}x + 4$

E. $y + 1 = -\frac{4}{6}(x + 5)$

F. $y + 1 = \frac{3}{2}(x + 5)$

2. The image shows a circle with center (4,6) and radius 10 units.



Select **all** points that lie on the circle.

A. $(-4,12)$

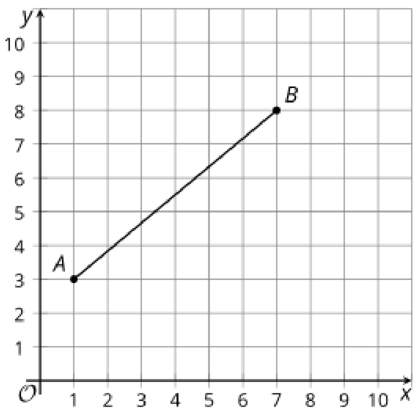
B. $(-6,6)$

C. $(4,6)$

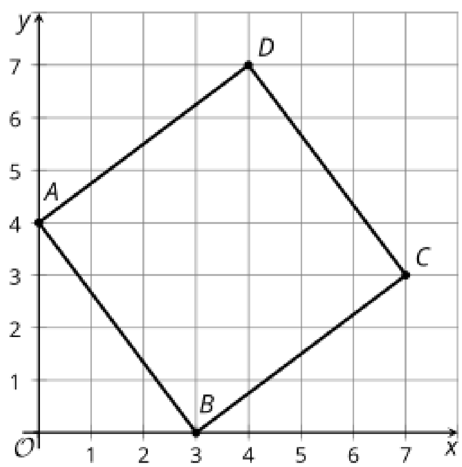
D. $(13,10)$

E. $(4,16)$

3. Find the point that partitions segment AB in a 2: 1 ratio.



4. The image shows quadrilateral $ABCD$.



a. Prove that quadrilateral $ABCD$ is a square.

b. Find the perimeter of $ABCD$. Explain or show your reasoning.

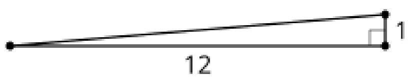
c. Find the area of $ABCD$. Explain or show your reasoning.

Lesson 15 Cumulative Practice Problems

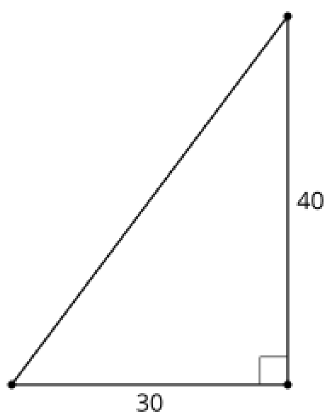
Finding sides in a right triangle

1. The Americans with Disabilities Act states that ramps must have an angle less than or equal to 4.8 degrees. A 4.8 degree angle in a right triangle has a 1: 12 ratio for the legs. Select **all** ramps that meet the Americans with Disabilities Act requirements.

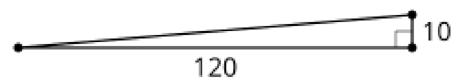
Triangle A



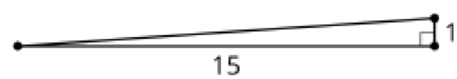
Triangle B



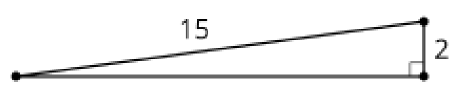
Triangle C



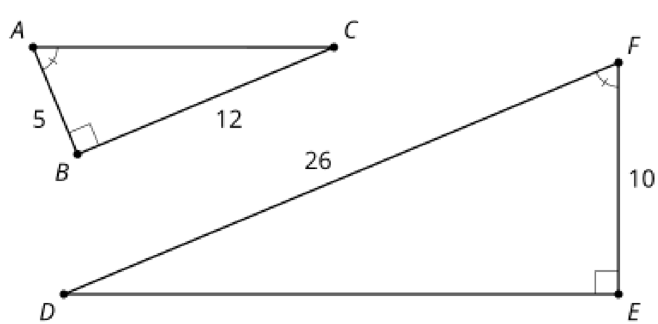
Triangle D



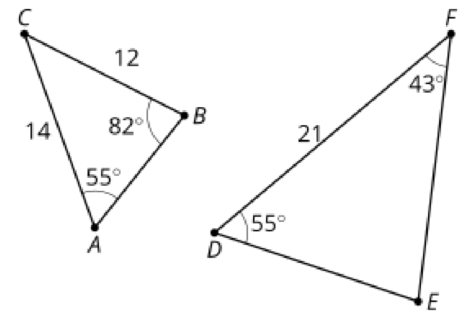
Triangle E



2. Find the missing side in each triangle using any method. Check your answers using a different method.



3. Use the triangles to answer the questions.



   a. Find the length of EF.

   b. Find the measure of angle E.

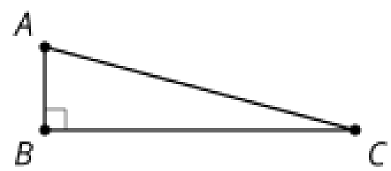
Finding sides in similar triangles

Lesson 16 Cumulative Practice Problems

The trigonometric ratios

1. Angle B is an acute angle in a right triangle. What is a reasonable approximation for angle B if the ratio for the opposite leg divided by the hypotenuse is 0.67?

2. Estimate the values to complete the table.



angle	adjacent leg ÷ hypotenuse	opposite leg ÷ hypotenuse	opposite leg ÷ adjacent leg
A			
C	0.97	0.26	0.27

3. Priya says, “I know everything about a right triangle with a 30 degree angle and a hypotenuse with length 1 cm. Here, look.“

   - The other angle is 60 degrees.
   - The leg adjacent to the 30 degree angle is 0.866 cm long.
   - The side opposite the 30 degree angle is 0.5 cm long.

   Han asks, “What would happen if a right triangle with a 30 degree angle has a hypotenuse that is 2 cm instead?“

   Help them find the missing angles and side lengths in the new triangle. Explain or show your reasoning.

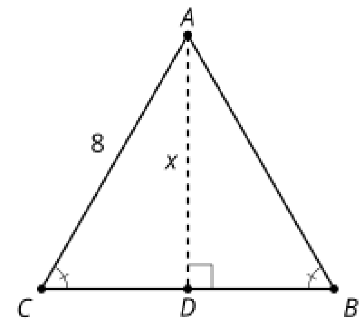
30-60-90

45-45-90

4. Triangle ABC is equilateral.

   a. What is the value of x?

   b. What is the measure of angle B?



5. An equilateral triangle has side length 8 units. What is the area?

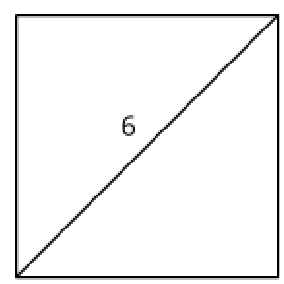
   A. $16\sqrt{3}$ square units

   B. 24 square units

   C. $24\sqrt{3}$ square units

   D. 32 square units

6. What is the length of the square's side?



   A. 3 units

   B. $\frac{6}{\sqrt{2}}$ units

   C. $6\sqrt{2}$ units

   D. 12 units

Running trig backwards

Lesson 17 Cumulative Practice Problems

1. Select **all** true statements:

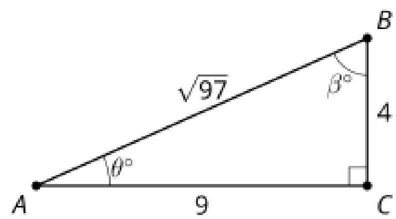
   A. $\sin(\theta) = \frac{4}{\sqrt{97}}$

   B. $\tan(\beta) = \frac{9}{4}$

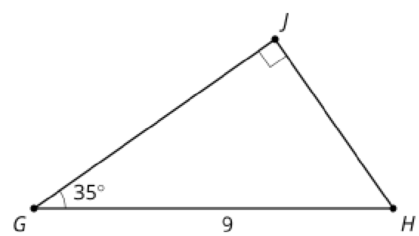
   C. $\tan(\beta) = \frac{4}{9}$

   D. $\cos(\beta) = \frac{4}{\sqrt{97}}$

   E. $4^2 + 9^2 = 97$



2. Write an expression that can be used to find the length of JH and an expression that can be used to find the length of GJ.



3. Andre and Clare are discussing triangle ABC that has a right angle at C and a hypotenuse of length 15 units. Andre thinks the triangle could have legs that are 9 and 12 units long. Clare thinks angle B could be 20 degrees and then side BC would be 14.1 units long. Do you agree with either of them? Explain or show your reasoning.

4. A triangle has sides with lengths 5, 12, and 13.

   a. Verify this is a Pythagorean triple.

   b. Approximate the acute angles in this triangle.

5. Approximate the angles that have the following quotients:

   a. adjacent leg ÷ hypotenuse = 0.966

   b. opposite leg ÷ hypotenuse = 0.469

   c. adjacent leg ÷ hypotenuse = 0.309

   d. opposite leg ÷ adjacent leg = 1.036

6. Lin missed class and Tyler is helping her use the table to approximate the angle measures that have the ratios listed. Tyler says, "You can use the right triangle table to figure this out." Lin notices that some of the ratios are the same in each row. Estimate the angles and explain why some of the values are repeated.

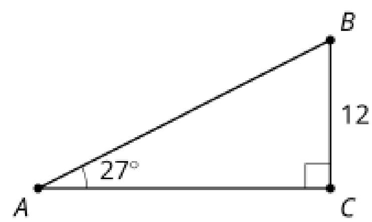
angle	**adjacent leg ÷ hypotenuse**	**opposite leg ÷ hypotenuse**	**opposite leg ÷ adjacent leg**
	0.573	0.819	1.428
	0.819	0.573	0.700

Lesson 18 Cumulative Practice Problems

1. *Technology required.* Mai is visiting Paris to see the Eiffel Tower. She is 80 feet away when she spots it. To see the top, she has to look up at an angle of 85.7 degrees. How tall is the Eiffel Tower?

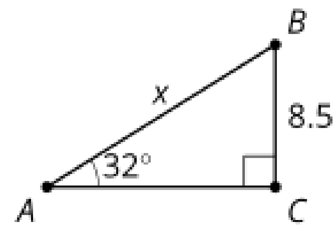
Solving trig story problems

2. *Technology required.* Find the missing measurements of the right triangle.



3. *Technology required.* Gateway Arch in St. Louis, Missouri, is 630 feet tall. Priya can look up at a 50 degree angle to see the top of the arch. How far away from the base of the arch is she standing?

4. Based on the figure, which equation is true?



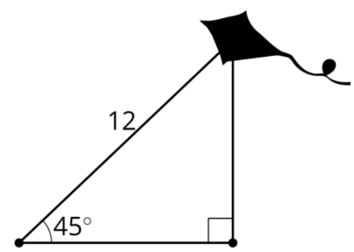
A. $\sin(32) = \frac{8.5}{x}$

B. $\sin(32) = \frac{x}{8.5}$

C. $\cos(32) = \frac{8.5}{x}$

D. $\cos(32) = \frac{x}{8.5}$

5. Kiran is flying a kite. He gets tired, so he stakes the kite into the ground. The kite is on a string that is 12 feet long and makes a 45 degree angle with the ground. How high is the kite?



A. 12 ft

B. $\frac{12}{\sqrt{2}}$ ft

C. $12\sqrt{2}$ ft

D. 24 ft

Lesson 14 Cumulative Practice Problems

Relationship between sin and cos

1. Select **all** the true equations.

   A. $\cos(15) = \sin(15)$

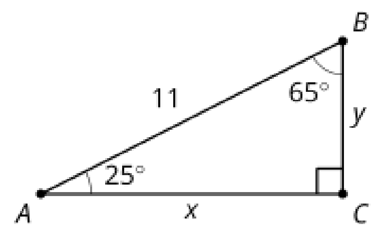
   B. $\cos(75) = \sin(15)$

   C. $\cos(75) = \cos(15)$

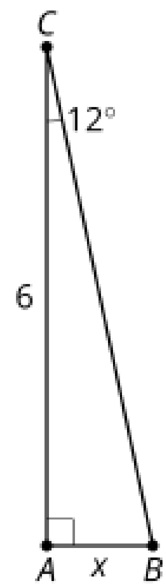
   D. $\cos(15) = \sin(75)$

   E. $\tan(15) = \tan(75)$

2. Write 2 expressions that can be used to find the value of x.

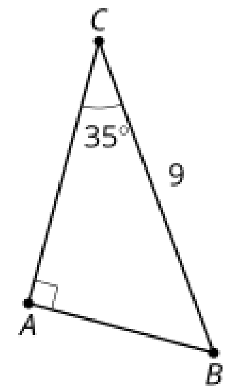


3. Andre and Mai are discussing how to solve for side AB. Andre thinks he can use the equation $\tan(12) = \frac{x}{6}$ to solve for AB. Mai thinks she can use the equation $\tan(78) = \frac{6}{x}$ to solve for AB. Do you agree with either of them? Show or explain your reasoning.



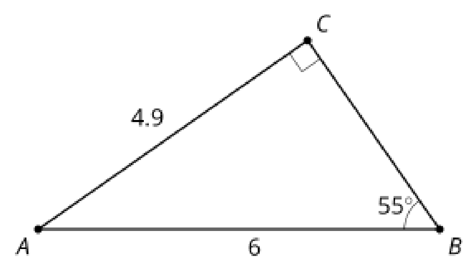
4. *Technology required.* Jada is visiting New York City to see the Empire State building. She is 100 feet away when she spots it. To see the top, she has to look up at an angle of 86.1 degrees. How tall is the Empire State building?

5. *Technology required.* Find the missing measurements in triangle ABC.



6. Right triangle ABC is shown.

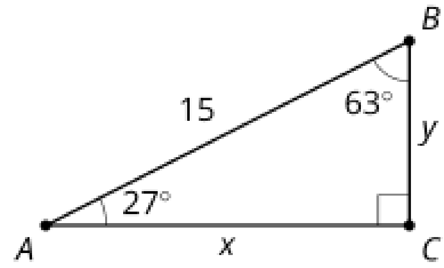
Write 2 expressions which are equal to the length of side BC.



Lesson 20 Cumulative Practice Problems - Review

Use videos from the last two weeks to assist you if you need help

1. Select **all** the true equations:



A. $\sin(27) = \frac{x}{15}$

B. $\cos(63) = \frac{y}{15}$

C. $\tan(27) = \frac{y}{x}$

D. $\sin(63) = \frac{x}{15}$

E. $\tan(63) = \frac{y}{x}$

2. What value of θ makes this equation true? $\sin(30) = \cos(\theta)$

A. -30

B. 30

C. 60

D. 180

3. A rope with a length of 3.5 meters is tied from a stake in the ground to the top of a tent. It forms a 17 degree angle with the ground. How tall is the tent?

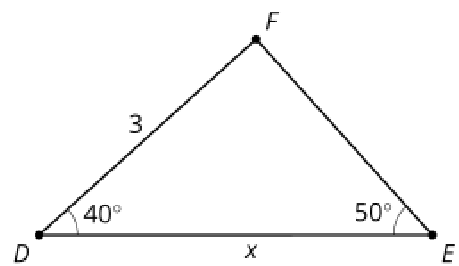
   A. 3.5tan(17)

   B. 3.5cos(17)

   C. 3.5sin(17)

   D. $\frac{\sin(17)}{3.5}$

4. *Technology required.* What is the value of x?



5. Find the missing side in each triangle using any method. Check your answers using a different method.